Characterization of log Fano varieties

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◇ My research interest

I am interested in birational geometry, in particular the minimal model theory and positive characteristics methods

◇ Research motivation

In this poster we consider the following problem:

Problem

Give a characterization of log Fano varieties

Definition[log Fano]

X: normal projective var.
X is *log Fano*
$\Leftrightarrow \exists \Delta$: eff. $\mathbb{Q}$-div. s.t (X, Δ) is klt and $-(K_X + \Delta)$ is ample.

This variety is important in the classification theory of algebraic varieties. For example, such a variety appears in the exceptional divisor of a divisorial contraction.

◇ Results

We work over the complex number field $\mathbb{C}$.

Theorem 1[Cascini-G, '13]

X: proj. var. with big $-K_X$,
Then X is log Fano
$\Leftrightarrow$ the anti-canonical ring $R(X, -K_X)$ is f.g. and
Proj $R(X, -K_X)$ has log terminal.

Theorem 2[GOST, KO, and Brown]

X: proj. var. with $h^1(X, \mathcal{O}_X) = 0$,
Then X is log Fano $\Leftrightarrow$ the Cox ring $\mathrm{Cox}(X)$ is f.g.
and Spec $\mathrm{Cox}(X)$ has log terminal.

First M. Brown showed "only if" part of the above theorem. And G–Okawa–Sannai–Takagi showed Theorem 2 by using modulo p method. And Kawamata–Okawa gives the another proof without modulo p method.

◇ log Fano vs Globally F-regular

The "if" part of Theorem 2 is actually an application of the following theorem:

Theorem [GOST]

X:Mori Dream Space. Then X is log Fano if it is of globally F-regular type.

Definition[Globally F-regular]

X: normal projective var. over an algebraic closed field of positive characteristic.
X is *Globally F-regular*
$\Leftrightarrow$ for $\forall D$: eff. div., $\exists e \in \mathbb{N}$ it holds that $f : \mathcal{O}_X \to F^e_*(\mathcal{O}_X(D))$ is splits as $\mathcal{O}_X$-module, where F is the Frobenius map and f is a composition of $F^e : \mathcal{O}_X \to F^e_*\mathcal{O}_X$ and $F^e_*\mathcal{O}_X \to F^e_*(\mathcal{O}_X(D))$.

Definition[Globally F-regular type]

X: complex normal projective var.
X is *of Globally F-regular*
$\Leftrightarrow$ the reduction modulo p model is globally F-regular

The above theorem also gives a partial answer of the following question:

Open Problem[Schwede–Smith]

X: complex projective normal var. Then X is log Fano if it is of globally F-regular type.