# 半対数的標準対の数値的自明な対数的標準因子に対するアバンダンス定理

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Conj.[Abundance conjecture]

**$X$: smooth proj. var. / $\mathbb{C}$.**

**If $K_X$ is nef, then $K_X$: semi-ample.**

## Definition

$X$: normal var. / $\mathbb{C}$,

$\Delta$: eff. $\mathbb{Q}$-divisor on $X$ s.t. $K_X + \Delta$: $\mathbb{Q}$-Car. div.

$\varphi : Y \longrightarrow X$: log res. of $(X, \Delta)$,

$$K_Y = \varphi^*(K_X + \Delta) + \sum a_i E_i$$

- $(X, \Delta)$: klt $\Leftrightarrow$ $a_i > -1$ $\forall i$,
- $(X, \Delta)$: lc $\Leftrightarrow$ $a_i \geq -1$ $\forall i$.

## Definition[Semi-log canonical]

$X$: red., $S_2$, pure dim., and n.c. in codim. $1$.
$\Delta \geq 0$: $\mathbb{Q}$-div. s.t. $K_X + \Delta$ is $\mathbb{Q}$-Car.
$X := \bigcup X_i$: irr. decomp,
$\nu : X' := \coprod X'_i \to X = \bigcup X_i$: normalization.
Define $\Theta$ by $K_{X'} + \Theta = \nu^*(K_X + \Delta)$.
$\Theta_i := \Theta|_{X'_i}$.
$(X, \Delta)$ is semi-log canonical (for short, slc) $\Leftrightarrow$ $(X'_i, \Theta_i)$: lc for every $i$.

Conj.[Log abundance conjecture]

**$(X, \Delta)$: proj. slc pair / $\mathbb{C}$.**

**If $K_X + \Delta$ is nef, then $K_X + \Delta$ is semi-ample.**

## Main theorem 1.

**$(X, \Delta)$: projective lc pair. Suppose that $K_X + \Delta \equiv 0$. Then $K_X + \Delta \sim_{\mathbb{Q}} 0$.**

## Main theorem 2.

**$(X, \Delta)$: projective slc pair. Suppose that $K_X + \Delta \equiv 0$. Then $K_X + \Delta \sim_{\mathbb{Q}} 0$.**

**Related results on Main Theorem 1:**

- $X$ has only canonical sing. by Kawamata, Tsunoda,
- $(X, \Delta)$ is klt by Nakayama, Ambro,
- $(X, \Delta)$ is lc by Campana–Koziarz–Paun, Kawamata (using Simpson's results).

**Our proof is independent of Simpson's results!**

**Related results on Main Theorem 2:**

- $\dim X \leq 2$ by Kawamata, Abramovich–Fong–Kollár–McKernan,
- $\dim X = 3$ by Fujino.

Theorem[G, 2009]

**$(X, \Delta)$: $d$-dim. lc weak log Fano pair. Suppose that $\mathrm{M}(X, \Delta) := \max\{\dim P | P$ :lc center of $(X, \Delta)\} \leq 1$. Then $-(K_X + \Delta)$: semi-ample.**

Theorem[cf. Fukuda, 2010]

**$(X, \Delta)$: $4$-dim. lc pair. Suppose that there exists a semi-ample divisor $D$ s.t. $K_X + \Delta \equiv D$. Then $-(K_X + \Delta)$: semi-ample.**