対数的標準特異点を持つ弱 Fano 多様体について

東京大学数理科学研究科
権業善範

2010 年 3 月 24 日

Definition

X: proj. normal var. / $\mathbb{C}$,
Δ: eff. $\mathbb{Q}$-divisor on X.
(X, Δ): weak log Fano pair $\Leftrightarrow$ $-(K_X + \Delta)$:nef & big $\mathbb{Q}$-Car.,
X: weak Fano var. $\Leftrightarrow$ $-K_X$:nef & big $\mathbb{Q}$-Car.

Definition

X: normal. var. / $\mathbb{C}$,

Δ: eff. $\mathbb{Q}$-divisor on X s.t. $K_X + \Delta$: $\mathbb{Q}$-Car. div.

$\varphi : Y \to X$: log res. of (X, Δ),

$$K_Y = \varphi^*(K_X + \Delta) + \sum a_i E_i$$

- (X, Δ): klt $\Leftrightarrow$ $a_i > -1 \ \forall i$,
- (X, Δ): lc $\Leftrightarrow$ $a_i \geq -1 \ \forall i$.

(X, Δ): lc weak log Fano pair.

Problem[Prokhorov, Shokurov]

(i) $-(K_X + \Delta)$: semiample?

(ii) $\overline{NE}(X)$: rational polyhedral?

They propose these problem on the study of "Complements on Surfaces".

Known results

(1) (X, Δ): **klt** $\Rightarrow$ **(i), (ii): O.K. by B.P.F.T.** $\&$ **Cone T.**

(2) **In** $\dim X = 2$, **Shokurov proved (2002).**

My studies are the above problems for "higher" dimensional "lc" weak log Fano pair.

My results are the following:

Result-1

In general, (i) & (ii) do not holds!

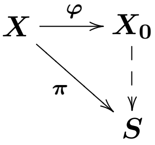
e.g.

S: very general 9-points blow up of $\mathbb{P}^2$,

$S \subset \mathbb{P}^N$:proj. normal emb.,

X_0: the proj. cone of $S \subset \mathbb{P}^{N+1}$,

$\varphi : X \to X_0$: the blow up at the vertex of X_0.



E: the exceptional divisor of φ.
Note that $E \simeq S$,

$$-(K_X + E)|_E = -K_E.$$

Because $-K_E$ is not semiample, $-(K_X + E)$ is not semiample.
We see that lc weak log Fano pair (X, E) does not satisfy (i)&(ii).

The 2nd results are the following:

Theorem

X: weak Fano 3-fold with lc sing.
Then $-K_X$ is semiample & $\overline{NE}(X)$ is rational polyhedral.

Theorem

X: weak Fano 4-fold with lc sing. Suppose that $\dim(\mathrm{Sing}(X)) \leq 1$ Then $-K_X$ is semiample & $\overline{NE}(X)$ is rational polyhedral.

Key point of the proof (3-dim. case)

(1) **We take ”good” dlt blow up s. t. alg. fib. sp. over any union of lc centers (by Fujino’s result),**

(2) **By the proof of B.P.F.T, we can reduce to lc centers,**

(3) **We must treat reducible schemes and alg. fib. sp. over seminormal curves,**

(4) **We patch the sections using the abundnace theorem for semi log canonical surfaces.**

Conj.[Abundance conjecture in the special case]

(X, Δ): projective slc pair. Suppose that $K_X + \Delta \equiv 0$. Then $K_X + \Delta \sim_{\mathbb{Q}} 0$.

the above conjecture is true in the following cases:

- $\dim X = 2$ by Kawamata, Abramovich–Fong–Kollár–McKernan,
- $\dim X = 3$ by Fujino.
- (X, Δ) is irreducible klt by Nakayama, Ambro.

For more higher dimensional, we can get the following:

Main result

Assume that Conj. in $(d-1)$-dim. holds.
(X, Δ): d-dim. lc weak log Fano pair. Suppose that
$M(X, \Delta) := \max\{\dim P | P$:lc center of $(X, \Delta)\} \leq 1$.
Then $-(K_X + \Delta)$: semiample.

In the next, we consider $\overline{NE}(X)$.

Theorem

(X, Δ): d-dim. lc weak log Fano pair. Suppose that $M(X, \Delta) := \max\{\dim P | P$:lc center of $(X, \Delta)\} \leq 1$.
Then $\overline{NE}(X)$: rational polyhedral cone.

The following theorem is the key theorem.

Cone theorem for arbitrary normal var.[Ambro, Fujino]

(X, Δ): **log pair. Then it holds that**

$$\overline{NE}(X) = \overline{NE}(X)_{K_X+\Delta\geq 0} + \overline{NE}(X)_{\mathrm{Nlc}(X,\Delta)} + \sum R_j,$$

where R_j: $(K_X + \Delta)$**-extremal ray and** $\{R_j\}$: **locally finite.**