Fréchet 空間上の quasi-regular non-local Dirichlet forms の 定式化と、その Φ_3^4 場の確率量子化への応用

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1 概要

Denote by S the Banach spaces of weighted real l^p, $1 \le p \le \infty$, spaces and the space of direct product $\mathbb{R}^\mathbb{N}$ (with $\mathbb{R}$ and resp. $\mathbb{N}$ the spaces of real numbers and resp. natural numbers), which are understood as Fréchet spaces. Let μ be a Borel probability measure on S. On the *real* $L^2(S; \mu)$ space, for each $0 < \alpha < 2$, we give an explicit formulation of α-stable type (cf., e.g., section 5 of [Fukushima,Uemura 2012] for corresponding formula on $L^2(\mathbb{R}^d)$, $d < \infty$) non-local quasi-regular (cf. section IV-3 of [M,R 92]) Dirichlet form $(\mathcal{E}_\alpha, \mathcal{D}(\mathcal{E}_\alpha))$ (with a domain $\mathcal{D}(\mathcal{E}_\alpha)$), and show an existence of S-valued Hunt processes properly associated to $(\mathcal{E}_\alpha, \mathcal{D}(\mathcal{E}_\alpha))$.

As an application of the above general results, we consider the problem of stochastic quantization of Euclidean free field, Φ_2^4 and Φ_3^4 fields, i.e., field with (self) interaction of 4-th power. By using the property that, for example, the support of the Euclidean Φ_3^4 field measure μ is in some *real* Hilbert space $\mathcal{H}_{-3}$, which is a sub space of the Schwartz space of *real* tempered distributions $\mathcal{S}'(\mathbb{R}^3 \to \mathbb{R})$, we define an isometric isomorphism $\tau_{-3} : \mathcal{H}_{-3} \to$ "some weighted l^2 space". By making use of τ_{-3}, we then interpret the above general theorems formulated on the abstract $L^2(S; \mu)$ space to the Euclidean Φ_3^4 field, $L^2(\mathcal{H}_{-3}; \mu)$, and for each $0 < \alpha \le 1$ we show the existence of an $\mathcal{H}_{-3}$-valued Hunt process $(Y_t)_{t \ge 0}$ the invariant measure of which is μ.

$(Y_t)_{t \ge 0}$ is understood as a stochastic quantization of Euclidean Φ_3^4 field realized by a Hunt process through the *non-local* Dirichlet form $(\mathcal{E}_\alpha, \mathcal{D}(\mathcal{E}_\alpha))$ for $0 < \alpha \le 1$.

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1)   As far as we know, there has been no explicit proposal of general formulation of *non-local* quasi-regular Dirichlet form on infinite dimensional topological vector spaces (for the local case, i.e., the case where the associated Markov processes are (continuous) diffusions, much have been developed and known), which admits interpretations to Dirichlet forms on several concrete random fields on several Fréchet spaces.

2)   Though there have been derived several results on the existence of (continuous) diffusions (i.e., roughly speaking, which associated to quadratic forms and generators of local type) corresponding with stochastic quantizations of Φ_2^4 or Φ_3^4 Euclidean fields (cf., the quotation given below), as far as we know, there exists no explicit corresponding consideration for *non-local* type Markov processes, which is performed through the Dirichlet form argument.

Hence, the present result is a first development that gives answers to the above mentioned open problems 1) and 2).

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