レヴィ市場におけるデジタルオプションに対する局所的リスク最小化問題について

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Locally risk-minimizing (LRM) is a well-known hedging method for contingent claims in a quadratic way (see e.g., [2] and [3]). By using Malliavin calculus, we can obtain explicit representations of LRM for incomplete market models whose asset price process is described by a solution to a stochastic differential equation (SDE) driven by a Lévy process ([1]).

On the other hand, there is one important derivative security describe by indicator function called digital option. A digital option pays a fixed cash amount if some condition is realized. Mathematical representation of digital (or binary) options are given by

$$\mathbf{1}_{[K,\infty)}(S_T) = \begin{cases} 1 & \text{for } S_T \geq K, \\ 0 & \text{otherwise,} \end{cases}$$

where $\{S_t\}_{t\in[0,T]}$ is a stock price process and $K > 0$ is a constant number that is fixed by the contract. It is popular and important derivative security. Therefore, to study digital options, we consider Malliavin differentiability of indicator functions ([4]).

In this talk, we first consider Malliavin differentiability of indicator functions on canonical Lévy spaces. By using it, we obtain explicit representations of LRM for digital options in markets driven by Lévy processes.

References

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