Biased random walk on the trace of biased random walk

D. A. Croydon (University of Warwick)

NB. This talk is based on the forthcoming work [8], which is joint with M. P. Holmes (University of Melbourne).

The study of stochastic processes in disordered media is an important aspect of modern probability. Models in this area for which extensive research has been conducted include the classical model of random walk in random environment, as well as random walks on random graphs, such as Galton-Watson trees and percolation clusters. Typical properties that one is interested in include: (i) recurrence/transience; (ii) laws of large numbers (i.e. the existence of a deterministic limiting velocity); (iii) conditions for ballisticity/sub-ballisticity (i.e. strict positivity of the speed or not); (iv) regularity (i.e. continuity, monotonicity or lack thereof) of attributes (e.g. the velocity) in terms of some underlying parameter; and (v) scaling limits.

In this paper, we tackle the first three of these issues for a biased random walk on a random graph, as given by the trace of a biased random walk on $\mathbb{Z}^d$; we henceforth call the process of interest the 'biased random walk on the trace of biased random walk' (BRWBRW). The biases are chosen so that both the original walk (defining the graph) and the BRWBRW are transient – somewhat remarkably, this does not mean that we necessarily require the underlying drift of the two walks to be oriented in the same direction. By standard regeneration arguments, the BRWBRW admits a limiting speed. Regarding the issue of ballisticity, we note that, when it backtracks, the initial walk creates traps for the BRWBRW. We will show that the effect of this trapping can lead to zero speeds, and in particular establish a sharp phase transition for whether the BRWBRW is ballistic or sub-ballistic.

We conclude by briefly relating our work to other studies in which trapping has been observed for biased random walk on random graphs. As early as the 1980s, physicists observed that such phenomenon might be relevant when the random graphs are percolation clusters, empirically demonstrating the non-monotonicity of the speed, and sub-ballisticity in the strong bias regime [1]. Mathematically, a phase transition between ballisticity and sub-ballisticity was first shown rigourously for the simpler model of random walk on supercritical Galton-Watson trees [10] (see also [3, 5, 7] for recent work concerning more detailed properties of such processes), and has since been confirmed to hold in the percolation setting [4, 9, 11]. A relatively up-to-date survey of these developments is given in [2]. Qualitatively, our results match those established for Galton-Watson trees and percolation clusters, and, although we do not confirm it rigourously, we also observe empirically non-monotonic behaviour for the speed that is similar to the behaviour expected for these other models. Moreover, whilst our graphs are more complex than trees, in the sense there is not a unique shortest path between vertices and the traps are less obviously defined, the model is still more tractable than the percolation case. As a result, we are able to give a more concrete expression for the critical point that separates the ballistic and sub-ballistic phase, which we are even able to evaluate explicitly in examples. Finally, we note that, in another related work, biased random walk on an unbiased random walk has been shown to exhibit localisation on a logarithmic scale [6].

References

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