

## Topics in Mathematical Science Report problem

NOTICE: Solve the following problems and submit your report to Division office of Mathematical Science (J613). The deadline is 12/17. S.Aida

Let us consider the two dimensional Gaussian space  $\Omega = \mathbb{R}^2$ ,  $\mathfrak{B} = \mathfrak{B}(\mathbb{R}^2)$  and

$$P(dw) = \frac{1}{\sqrt{2\pi}^2} \exp\left(-\frac{1}{2}(w_1^2 + w_2^2)\right) dw_1 dw_2.$$

Let  $X(w) = e^{w_1+w_2}$ .

(1) Show that  $X(w)$  satisfies the assumptions in Theorem 4.

(2) Show that the density function of the law of  $X(w) = e^{w_1+w_2}$  is

$$\rho(x) = \frac{1}{\sqrt{4\pi}} \frac{1}{x} \exp\left(-\frac{(\log x)^2}{4}\right) \quad (x > 0).$$

Hint: Show that for any bounded continuous function  $\varphi$  on  $\mathbb{R}$

$$\int_{\mathbb{R}^2} \varphi(e^{w_1+w_2}) P(dw) = \int_{\mathbb{R}} \varphi(x) \frac{1}{\sqrt{4\pi}} \frac{1}{x} \exp\left(-\frac{(\log x)^2}{4}\right) dx.$$

Remark: The law of  $X$  is called a log-normal distribution.