K-STABILITY AND CM-STABILITY FOR FANO MANIFOLDS

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ABSTRACT

In [3], Tian introduced two concepts of “stability” for Fano manifolds, i.e., K-stability and CM-stability, and proved the following fact:

Fact 1. If a Fano manifold $X$ admits an Einstein-Kähler metric and the holomorphic automorphism group of $X$ is finite, then $X$ is both K-stable and CM-stable.

Inspired by Hilbert-Mumford’s numerical criterion for the stability in the geometric invariant theory ([2]), Tian conjectured that the K-stability and the CM-stability are equivalent. For Fano hypersurfaces in $\mathbb{P}^{n+1}(\mathbb{C})$, this equivalence was proved by Bauer ([1]).

In this talk, we will report on the equivalence of the K-stability and the CM-stability for the general Fano manifolds, that is, if we change slightly the definitions of the K-stability and the CM-stability, then we can prove the following theorem under some assumption.

Theorem 1. Let $X$ be a Fano manifold. Then:

(i) $X$ is K-semistable if and only if $X$ is CM-semistable.
(ii) $X$ is K-stable if and only if $X$ is CM-stable.

REFERENCES


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