

# Trends in Modern Geometry

July 10th (Mon) – 13th (Thu), 2017

Lecture Hall, Graduate School of Mathematical Sciences, the University of Tokyo

## July 10th (Monday)

8:00–9:00 Registration

9:00–9:50 Shouhei Honda (Tohoku U)

“New stability results for sequences of metric measure spaces with uniform Ricci bounds from below”

10:30–11:20 Yuji Sano (Fukuoka U)

“A moment map picture of relative balanced metrics on extremal Kähler manifolds”

12:00–12:50 Yuji Odaka (Kyoto U)

“Tropical geometric compactification of moduli”

15:00–15:50 Jian Song (Rutgers U)

“Degeneration of Kähler-Einstein manifolds with negative scalar curvature”

16:30–17:20 Matthew J. Gursky (U Notre Dame)

“Local existence and non-existence results for Poincaré-Einstein metrics”

## July 11th (Tuesday)

9:00–9:50 Jean-Yves Welschinger (U Lyon1)

“Expected topology of a random subcomplex in a simplicial complex”

10:30–11:20 Michael Usher (U Georgia)

“Existence and uniqueness for symplectic embeddings”

12:00–12:50 Kwokwai Chan (CU Hong Kong)

“Scattering diagrams and Maurer-Cartan equations”

15:00–15:50 Hiroshi Ohta (Nagoya U)

“Complete partition and application to autoequivalences”

16:30–17:20 Simon Brendle (Columbia U)

“Ricci flow in higher dimensions”

18:00–21:00 Reception

**July 12th (Wednesday)**

9:00–9:50 Jean-Pierre Demailly (U Grenoble)

“General extension theorem for cohomology classes on non reduced analytic subspaces”

10:30–11:20 Sébastien Boucksom (l'École Polytechnique)

“Determinant of cohomology and Deligne pairings in complex and non-Archimedean geometry”

12:00–12:50 Tamás Darvas (U Maryland)

“Complex Monge-Ampere equations with prescribed singularity type”

15:00–15:50 Ved Datar (UC Berkeley)

“Extremal metrics on blow-ups”

16:30–17:20 Jeff Streets (UC Irvine)

“Generalized Kähler Ricci flow and a generalized Calabi conjecture”

**July 13th (Thursday)**

9:00–9:50 Huai-Dong Cao (Lehigh U)

“Geometry of Ricci Solitons”

10:15–11:05 Hajime Ono (Saitama U)

“Conformally Kaehler Einstein-Maxwell metrics and volume minimization”

11:30–12:20 Jeff Viaclovsky (UC Irvine)

“Scalar-flat Kähler ALE metrics”

## Abstract

### July 10th (Monday)

- 9:00–9:50 Shouhei Honda (Tohoku U)  
“New stability results for sequences of metric measure spaces with uniform Ricci bounds from below”  
In this talk we introduce several stability results with respect to the measured Gromov-Hausdorff convergence under assuming uniform lower bounds on Ricci curvature. In particular we prove stability of Sobolev/BV functions, Hessians, and generalized Bakry-Emery conditions. Applications include a suspension theorem with positive Ricci curvature, which is new even for smooth manifolds. More recent work is also discussed if there is sufficient time. This is a joint work with Luigi Ambrosio.
- 10:30–11:20 Yuji Sano (Fukuoka U)  
“A moment map picture of relative balanced metrics on extremal Kähler manifolds”  
Donaldson gave a general framework to study some specific Fubini-Study metrics on a polarized manifold, which are called balanced metrics. As an application, he gave the quantization of the constant scalar curvature metrics by balanced metrics. In this talk, I explain the extension of Donaldson’s framework to some relative balanced metrics and its application to the extremal Kähler metrics. This is a joint work with Carl Tipler.
- 12:00–12:50 Yuji Odaka (Kyoto U)  
“Tropical geometric compactification of moduli”  
I will describe certain compactifications of the moduli spaces of e.g., algebraic curves, abelian varieties, K3 surfaces etc, all polarized, which I call as in the title. The original moduli spaces are mostly classical complex analytic moduli spaces, while the attached boundaries are “tropical” (Not varieties), parametrizing certain “tropical varieties”. This is started in our arXiv:1406.7772, continued in 1705.05545, further developed with Y.Oshima (in preparation), and still ongoing. If time’d remain, I might end the talk by making some irrelevant remarks and pose problems on the framework of K-stability, partially to “unify” the usual (global) K-stability and C.Li’s nice normalization of volume for klt singularities. cf., my arXiv:1411.5487 and Chi Li’s talk.

- 15:00–15:50 Jian Song (Rutgers U)

“Degeneration of Kähler-Einstein manifolds with negative scalar curvature”

We consider any algebraic family of Kähler-Einstein manifolds of negative scalar curvature over a punctured disc  $B \setminus \{0\}$ . We show that the Kähler-Einstein manifolds converge, as the parameter of the punctured disc tends to 0, in pointed Gromov-Hausdorff topology to a unique finite disjoint union of complete metric length spaces homeomorphic to a projective semi-log canonical model with its locus of non log-terminal singularities removed. In particular, the limiting metric is a smooth Kähler-Einstein metric on the nonsingular part of the semi-log canonical model and the Hausdorff measure of the limit metric space is equal to the volume of Kähler-Einstein manifolds. This is the first step of our approach toward compactification of the analytic geometric moduli space of Kähler-Einstein manifolds of negative first Chern class.

- 16:30–17:20 Matthew J. Gursky (U Notre Dame)

“Local existence and non-existence results for Poincaré-Einstein metrics”

I will begin with a brief overview of the existence question for conformally compact Einstein manifolds with prescribed conformal infinity. After stating the seminal result of Graham-Lee, I will discuss a non-existence result (joint with Qing Han) for certain conformal classes on the 7-dimensional sphere. I will also mention work (with Gabor Székelyhidi) on a version of “local existence” of Poincaré-Einstein metrics.

## July 11th (Tuesday)

- 9:00–9:50 9:00–9:50 Jean-Yves Welschinger (U Lyon1)

“Expected topology of a random subcomplex in a simplicial complex”

I will explain how to bound from above and below the expected Betti numbers of a random subcomplex in a simplicial complex and get asymptotic results under infinitely many barycentric subdivisions. This is a joint work with Nermin Salepci. It complements previous joint works with Damien Gayet on random topology.

- 10:30–11:20 Michael Usher (U Georgia)

“Existence and uniqueness for symplectic embeddings”

Among the most famous early results in symplectic topology is Gromov’s non-squeezing theorem, asserting that if a ball can be symplectically embedded into a

cylinder then the radius of the cylinder must be at least the radius of the ball. Since then, the more general problem of when one region in  $R^{2n}$  can be symplectically embedded into another has been well-studied, and especially in dimension four a great deal is now known, with the answers to obvious questions turning out to be remarkably intricate and involving connections to complex algebraic geometry and elementary number theory. It is also natural to ask whether embeddings from one region into another are unique up to a suitable notion of equivalence, such as composition with symplectomorphisms of the target; McDuff has shown that this is the case when both the domain and the codomain are four-dimensional ellipsoids. I will survey results relating both to existence and uniqueness of symplectic embeddings between regions in  $R^4$ , and explain recent joint work with Jean Gutt which shows how proving existence for one embedding problem often gives rise to non-uniqueness for another embedding problem. In particular we show that the analogue of McDuff's theorem is false for certain regions that are arbitrarily close to ellipsoids.

- 12:00–12:50 Kwokwai Chan (CU Hong Kong)

“Scattering diagrams and Maurer-Cartan equations”

In 2005, a program was set forth by Fukaya aiming at investigating SYZ mirror symmetry by asymptotic analysis on Maurer-Cartan equations. In this talk, I will explain some results which implement a half of Fukaya's program. More precisely, I will show how semi-classical limits of Maurer-Cartan solutions give rise naturally to consistent scattering diagrams, which are known to encode Gromov-Witten data on the mirror side and have played an important role in the works of Kontsevich-Soibelman and Gross-Siebert on the reconstruction problem in mirror symmetry. This talk is based on joint work with Conan Leung and Ziming Ma, which was substantially supported by a grant from the Research Grants Council of the Hong Kong Special Administrative Region, China (Project No. CUHK14302015).

- 15:00–15:50 Hiroshi Ohta (Nagoya U)

“Complete partition and application to autoequivalences”

I will discuss subgroups generated by spherical twists in the autoequivalence group of a triangulated category, with some geometric application to Fukaya category. This is based on a joint work in progress with Jongmyeong Kim.

- 16:30–17:20 Simon Brendle (Columbia U)

“Ricci flow in higher dimensions”

The Ricci flow, introduced in 1982 by Richard Hamilton, has become one of the most important tools for studying the geometry and topology of manifolds. In dimension 3, we have a good understanding of how singularities form thanks to the work of Hamilton and Perelman. In this lecture, I will discuss how results of a similar nature can be obtained in higher dimensions, under suitable restrictions on the curvature of the initial metric. As a corollary, we obtain a classification, up to diffeomorphism, of all manifolds satisfying these curvature conditions. Our assumptions are sharp in many interesting geometric situations, such as for projective spaces and products of spheres.

### July 12th (Wednesday)

- Jean-Pierre Demailly (U Grenoble)

“General extension theorem for cohomology classes on non reduced analytic subspaces”

The main purpose of the talk is to generalize the celebrated  $L^2$  extension theorem of Ohsawa-Takegoshi in several new directions : the holomorphic sections to extend are taken in a possibly singular hermitian line bundle, the subvariety from which the extension is performed may be non reduced, the ambient manifold is Kähler and holomorphically convex.

- 10:30–11:20 Sébastien Boucksom (l'École Polytechnique)

“Determinant of cohomology and Deligne pairings in complex and non-Archimedean geometry”

The determinant of cohomology of a line bundle on a complex projective manifold can be seen as a complex line-valued analogue of the Euler characteristic. It admits a polynomial Riemann-Roch-like expansion, both sides of which can be naturally equipped with metrics, following a program initiated by Deligne. We will discuss the close relation of this construction with Bergman kernel asymptotics, as well as a version of it in the setting of Berkovich spaces, building upon recent developments in non-Archimedean pluripotential theory.

- 12:00–12:50 Tamás Darvas (U Maryland)

“Complex Monge-Ampere equations with prescribed singularity type”

Given a Kähler manifold  $(X, \omega)$ , finding smooth solutions to the equation  $(\omega +$

$i\partial\bar{\partial}u)^n = f\omega^n$  goes back to Yau’s solution of the Calabi conjecture in the seventies. In joint work with E. Di Nezza and C.H. Lu, we proposed to solve this same equation with the added constraint that  $u \in PSH(X, \omega)$  has prescribed singularity type. As it turns out, this problem is well posed only for a certain class of (model) singularity types that we characterize, and we also solve the corresponding equation. Our results extend to the case of big cohomology classes as well.

- 15:00–15:50 Ved Datar (UC Berkeley)

“Extremal metrics on blow-ups”

It is known that the blow-up of a constant scalar curvature Kähler manifold of dimension higher than 2 also admits a metric of constant scalar curvature if it is K-stable. The analogous question relating extremal metrics on the blow-up to relative stability is still open. I will talk about some of these open problems and present some partial results.

- 16:30–17:20 Jeff Streets (UC Irvine)

“Generalized Kähler Ricci flow and a generalized Calabi conjecture”

Generalized Kähler geometry is a natural extension of Kähler geometry with roots in mathematical physics, and is a particularly rich instance of Hitchin’s program of ‘generalized geometries.’ In this talk I will discuss an extension of Kähler-Ricci flow to this setting. I will formulate a natural Calabi-Yau type conjecture based on Hitchin/Gualtieri’s definition of generalized Calabi-Yau equations, then introduce the flow as a tool for resolving this conjecture. The main result is a global existence and convergence result for the flow which yields a partial resolution, and which classifies generalized Kähler structures on hyperKähler backgrounds.

### July 13rd (Thursday)

- 9:00–9:50 Huai-Dong Cao (Lehigh U)

“Geometry of Ricci Solitons”

Ricci solitons are self-similar solutions to the Ricci flow which model singularity formations. They are natural extensions of Einstein metrics, and are also critical points of certain functionals defined by Perelman and others. In this talk I shall survey some of the recent developments on Ricci solitons.

- 10:15–11:05 Hajime Ono (Saitama U)

“Conformally Kaehler Einstein-Maxwell metrics and volume minimization”

Apostolov and Maschler introduced the notion of conformally Kaehler Einstein-Maxwell metrics (cKEM metrics for short), which is a generalization of strongly Hermitian solutions of the Einstein-Maxwell equations on complex surfaces. Moreover they defined an obstruction to the existence of cKEM metrics, cKEM-Futaki invariants. In this talk, I would like to explain that cKEM-Futaki invariants are considered as the first variation of certain volume functional. We also give some examples of non-Kaehler cKEM metrics in any dimensions. This talk is based on a joint work with A. Futaki.

- 11:30–12:20 Jeff Viaclovsky (UC Irvine)

“Scalar-flat Kähler ALE metrics”

I will discuss the local structure of the moduli space of scalar-flat Kähler asymptotically locally Euclidean metrics, compactness theorems, and some existence results.