Trends in Modern Geometry

July 21st (Thu) –24th (Sun), 2016

Lecture Hall, Graduate School of Mathematical Sciences, the University of Tokyo

July 21st (Thursday)

8:00–9:00 Registration

9:00–9:50 Kento Fujita (Kyoto, RIMS)

"A valuative criterion for uniform K-stability of Fano manifolds"

10:15–11:05 Cristiano Spotti (Cambridge)

"Resolutions of conically singular cscK varieties"

11:30–12:20 Georges Dloussky (Aix-Marseille Univ)

"Locally conformally symplectic structures on compact non-Kähler complex surfaces"

14:30–15:20 Ayato Mitsuishi (Gakushuin)

"Orientabilities and fundamental classes of Alexandrov spaces and its applications"

16:00–16:50 Jeff Viaclovsky (Wisconsin, Madison)

"Deformation theory of scalar-flat Kähler ALE surfaces"

July 22nd (Friday)

9:00–9:50 Egor Shelukhin (Princeton, IAS)

"Measurements of transformations in symplectic topology"

10:30–11:20 Jake Solomon (Hebrew Univ.)

"The space of positive Lagrangians"

12:00–12:50 Kaoru Ono (Kyoto, RIMS)

"Lagrangian Floer theory and Generation Criterion for Fukaya category"

15:00–15:50 Yoshinori Hashimoto (Univ College London)

"Quantisation of extremal Kähler metrics"

16:30–17:20 Christopher Woodward (Rutgers)

"Lagrangian submanifolds and minimal model transitions"

July 23rd (Saturday)

9:00–9:50 Tomoyuki Hisamoto (Nagoya)
  "Stability and coercivity for toric polarized manifolds"

10:30–11:20 Kento Fujita (Kyoto, RIMS)
  "K-stability of Fano manifolds with not small alpha invariants'

12:00–12:50 Richard Bamler (Berkeley)
  "Convergence of Ricci flows with bounded scalar curvature"

15:00–15:50 Ursula Hamenstaedt (Bonn)
  "Geometric invariant of closed hyperbolic 3-manifolds"

16:30–17:20 John Lott (Berkeley)
  "Ricci flow through singularities"

18:00– Reception at Common Room on the 2nd floor of Mathematical Sciences Building.

July 24th (Sunday)

9:00–9:50 Ronan Conlon (L'Univ du Québec à Montréal)
  "New examples of gradient expanding Kähler-Ricci solitons"

10:30–11:20 Tristan Collins (Harvard)
  "Sasaki-Einstein metrics and K-Stability"

12:00–12:50 Claude LeBrun (Stony Brook)
  "Mass in Kähler Geometry"

Abstract

July 21st (Thursday)

- 9:00–9:50 Kento Fujita (Kyoto, RIMS)

  "A valuative criterion for uniform K-stability of Fano manifolds"

  It is an interesting problem whether a given Fano manifold admits Kähler-Einstein metrics or not. It has been known recently that this condition is equivalent to the condition "K-polystability" , which is purely algebraic. In this talk, we mainly treat uniform K-stability of Fano manifolds, which is stronger than K-polystability. More precisely, we will give a simple necessary and sufficient condition for uniform K-stability of Fano manifolds.

- 10:15–11:05 Cristiano Spotti (Cambridge)

  "Resolutions of conically singular cscK varieties"

  I will describe a construction of Kähler metrics with constant scalar curvature (cscK) on certain crepant resolutions of cscK varieties with isolated singularities modelled on Calabi-Yau cones. This is joint work with C. Arezzo.

- 11:30–12:20 Georges Dloussky (Aix-Marseille Univ)

  "Locally conformally symplectic structures on compact non-Kähler complex surfaces"

  Joint work with Vestislav Apostolov. We show that all compact surfaces with odd first Betti number admit locally conformally symplectic structures taming the complex structure (even for possible unknown surfaces in class $VII0$). We introduce and study the subset $C(S)$ (respectively, $T(S)$) in $H^1_{dR}(M)$ of classes a for which there exists an lcK metric on S with Lee form $\theta \in a$ (respectively, for which there exists a locally conformally symplectic form which tames J, with Lee form in a). arXiv:1501.02687.

- 14:30–15:20 Ayato Mitsuishi (Gakushuin)

  "Orientabilities and fundamental classes of Alexandrov spaces and its applications"

  Alexandrov spaces are complete metric spaces with a lower curvature bound in the sense that any geodesic triangle is not thinner than a model surface of constant curvature. Such a space naturally appears as the Gromov-Hausdorff limit of complete Riemannian manifolds whose sectional curvature are uniformly bounded from below. The orientability is very fundamental concept for manifolds. For

Alexandrov spaces, the notions of orientability were considered in several ways. I will announce that such notions are equivalent. Further, I will give several applications to this result.

- 16:00–16:50 Jeff Viaclovsky (Wisconsin, Madison)
  "Deformation theory of scalar-flat Kähler ALE surfaces"
  I will discuss a Kuranishi-type theorem for deformations of complex structure on ALE Kähler surfaces, which will be used to prove that for any scalar-flat Kähler ALE surface, all small deformations of complex structure also admit scalar-flat Kähler ALE metrics. A local moduli space of scalar-flat Kähler ALE metrics can then be constructed which is universal up to small diffeomorphisms. I will also discuss a formula for the dimension of the local moduli space in the case of a scalar-flat Kähler ALE surface which deforms to a minimal resolution of an isolated quotient singularity. This is joint work with Jiyuan Han.

July 22nd (Friday)

- 9:00–9:50 Egor Shelukhin (Princeton, IAS)
  "Measurements of transformations in symplectic topology"
  We discuss applications of quasi-morphisms and other novel tools to various ways of measuring distance between natural transformations in symplectic topology. Time permitting, we discuss recent applications of pesistent homology in this direction, and analogues of these investigations in contact topology.

- 10:30–11:20 Jake Solomon (Hebrew Univ.)
  "The space of positive Lagrangians"
  Specifically, a Hamiltonian isotopy class of positive Lagrangians admits a Riemannian metric of non-positive sectional curvature and a convex functional which has critical points at special Lagrangians. Geodesics are equivalent to solutions of the degenerate special Lagrangian equation. Existence of geodesics would imply uniqueness of special Lagrangians as well as a version of the strong Arnold conjecture. Weak geodesics are known to exist between positive graph Lagrangians in Euclidean space. Smooth geodesics can be constructed in Milnor fibers and toric Calabi-Yau manifolds using symmetry techniques. This talk is based partially on joint work with Y. Rubinstein and A. Yuval.

- 12:00–12:50 Kaoru Ono (Kyoto, RIMS)

  "Lagrangian Floer theory and Generation Criterion for Fukaya category"

  Based on joint works with K. Fukaya, Y.-G. Oh and H. Ohta and with M. Abouzaid, K. Fukaya, Y.-G. Oh, H. Ohta, I plan to explain several constructions in Lagrangian Floer theory related to generation criterion for Fukaya category.

- 15:00–15:50 Yoshinori Hashimoto (Univ College London)

  "Quantisation of extremal Kähler metrics"

  Kähler metrics with "optimal" curvature properties, such as constant scalar curvature Kähler (cscK) or Kähler-Einstein metrics, have been studied intensively in recent years. Extremal Kähler metrics, as proposed by Calabi, generalise these classes of metrics and give a unified treatment of them. It is well-known that a foundational theorem called Donaldson's quantisation provides "finite dimensional" approximation of cscK metrics and insight into GIT (Geometric Invariant Theory) stability properties of the underlying manifold if the automorphism group is discrete, but this theorem does not naively extend to the case where the automorphism group is non-discrete. We propose in this talk a new "quantising" equation, which generalises various key results in Donaldson's quantisation when the automorphism group is no longer discrete, and can be applied more generally to extremal Kähler metrics.

- 16:30–17:20 Christopher Woodward (Rutgers)

  "Lagrangian submanifolds and minimal model transitions"

  The Fukaya category of Lagrangian submanifolds of a symplectic manifold is supposed to play a role in the homological mirror symmetry conjecture. But generators for Fukaya categories are known in only very few examples. I will describe a method for finding generators by associating Lagrangians to minimal model program (conjecturally equivalent to Kahler-Ricci flow with surgery) which "organizes" some results of Fukaya-Oh-Ohta-Ono on toric varieties, and works well for more general examples such as moduli spaces of genus zero parabolic bundles or stable marked curves. Partly joint with Francois Charest and Sushmita Venugopalan.

July 23rd (Saturday)

- 9:00–9:50 Tomoyuki Hisamoto (Nagoya)

  "Stability and coercivity for toric polarized manifolds"

  We introduce "J-uniform K-polystability" for toric polarizations and show that it is equivalent to the natural growth condition of the K-energy.

- 10:30–11:20 Kento Fujita (Kyoto, RIMS)

  "K-stability of Fano manifolds with not small alpha invariants'

  I want to show in this talk that any n-dimensional Fano manifold X with $\alpha(X) = n/(n+1)$ and $n \geq 2$ is K-stable, where $\alpha(X)$ is the alpha invariant of X introduced by Tian. In particular, any such X admits Kähler-Einstein metrics and the holomorphic automorphism group of X is finite.

- 12:00–12:50 Richard Bamler (Berkeley)

  "Convergence of Ricci flows with bounded scalar curvature"

  It is a basic fact that the Riemannian curvature becomes unbounded at every finite-time singularity of the Ricci flow. Sesum showed that the same is true for the Ricci curvature. It has since remained a conjecture whether also the scalar curvature becomes unbounded at any singular time. In this talk I will show that, given a uniform scalar curvature bound, the Ricci flow can only degenerate on a set of codimension bigger or equal to 4, if at all. This result is a consequence of a structure theory for such Ricci flows, which relies on and generalizes recent work of Cheeger and Naber.

- 15:00–15:50 Ursula Hamenstaedt (Bonn)

  "Geometric invariant of closed hyperbolic 3-manifolds"

  Perelman's solution of the geometrization conjecture implies that "most" closed 3-manifold admit a hyperbolic metric. This notion of "most" can be made precise using a notion of randomness for 3-manifolds. I will describe recent results on the spectrum of the Laplace operator for such manifolds which are roughly sharp and distinguish random manifolds. I will use this to provided evidence for the validity of recent conjectures on the structure of such manifolds in the random case.

- 16:30–17:20 John Lott (Berkeley)

  "Ricci flow through singularities"

  Perelman's Ricci flow-with-surgery involves a surgery parameter δ, which describes the scale at which surgery is performed. We show that there is a subse-

quential limit as δ goes to zero, thereby partially answering a question of Perelman. The limiting object is called a singular Ricci flow. Such objects can be considered to be flows through singularities, and studied in their own right. We prove some geometric and analytical properties of such singular Ricci flows. This is joint work with Bruce Kleiner.

July 24th (Sunday)

- 9:00–9:50 Ronan Conlon (L'Univ du Québec à Montréal)

  "New examples of gradient expanding Kähler-Ricci solitons"

  A complete Kähler metric g on a Kähler manifold M is a *gradient expanding Kähler-Ricci soliton* if there exists a smooth real-valued function $f : M \to \mathbb{R}$ with $\nabla^g f$ holomorphic such that $\mathrm{Ric}(g) - \mathrm{Hess}(f) + g = 0$. I will present new examples of such metrics on the total space of certain holomorphic line bundles. This is joint work with Alix Deruelle (Université Paris-Sud).

- 10:30–11:20 Tristan Collins (Harvard)

  "Sasaki-Einstein metrics and K-Stability"

  I will discuss the connection between Sasaki-Einstein metrics, or conical Ricci flat Kähler metrics, and the algebro-geometric notion of K-stability, and some applications to finding new Einstein metrics in the 5-sphere.

- 12:00–12:50 Claude LeBrun (Stony Brook)

  "Mass in Kähler Geometry"

  Given a complete Riemannian manifold that looks enough like Euclidean space at infinity, physicists have defined a quantity called the "mass" which measures the asymptotic deviation of the geometry from the Euclidean model. In this lecture, I will explain a simple formula, discovered in joint work with Hajo Hein, for the mass of any asymptotically locally Euclidean (ALE) Kähler manifold. For ALE scalar-flat Kähler manifolds, the mass turns out to be a topological invariant, depending only on the underlying smooth manifold, the first Chern class of the complex structure, and the Kähler class of the metric. When the metric is actually AE (asymptotically Euclidean), our formula not only implies a positive mass theorem for Kähler metrics, but also yields a Penrose-type inequality for the mass.