Trends in Modern Geometry

July 7th (Mon) –11st (Fri), 2014

Lecture Hall, Graduate School of Mathematical Sciences, the University of Tokyo

July 7th (Monday)

14:00–15:00 Xiuxiong Chen (Stony Brook)

"On the Kähler Ricci flow"

15:30–16:30 Song Sun (Stony Brook)

"Kähler-Einstein metrics and Gromov-Hausdorff limits"

July 8th (Tuesday)

9:30–10:30 Dietmar Salamon (ETH)

"A differential geometric approach to GIT"

11:00–12:00 Jian Song (Rutgers)

"Riemannian geometry of Kähler-Einstein currents"

14:00–15:00 Shouhei Honda (Kyushu)

"Cheeger constant, p-Laplacian, and Gromov-Hausdorff convergence"

15:30–16:30 Takao Yamaguchi (Kyoto)

"Lipschitz homotpy structure of Alexandrov spaces"

July 9th (Wednesday)

9:30–10:30 Olivier Biquard (ENS)

"Einstein metrics, complex singularities, and the AH problem"

11:00–12:00 Naichung Conan Leung (CUHK)

"Gradient flow trees from Witten deformation"

July 10th (Thursday)

9:30–10:30 Claudio Arezzo (ICTP)

"Constant Scalar Curvature Kähler metrics on Resolutions of singularities"

11:00–12:00 Sean Paul (Wisconsin-Madison)

"Introduction to semistable pairs and applications"

13:30–14:30 Ngaiming Mok (Hong Kong)

"Geometric structures and substructures on uniruled projective manifolds"

15:00–16:00 Kenji Fukaya (SCGP)

"Sheaf of perturbation data in de Rham version of virtual fundamental chain"

16:30–17:30 Gang Tian (Beijing/Princeton)

"An extension of Matsushima Theorem"

18:00– Reception

July 11st (Friday)

9:30–10:30 Yuji Sano (Kumamoto)

"On the extremal vector fields on smooth toric Fano manifolds"

11:00–12:00 Kaoru Ono (RIMS)

"Floer complex and covering spaces"

Abstract

July 7th (Monday)

- 14:00–15:00 Xiuxiong Chen (Stony Brook)
  "On the Kähler Ricci flow"
  This is a joint work with B. Wang. Based on the compactness of the moduli space of non-collapse Calabi Yau space with mild singularities. Our theory is a generalization of the well known theory of non-collapsed Kähler Einstein manifolds. As an application, we prove the Hamilton Tian conjecture on the Kähler Ricci flow, as well as the complete partial C^0 estimate of Tian for Kähler metrics with Ricci bounded from below. Donaldson-Sun proved the partial C^0 estimate for Kähler Einstein metrics.

- 15:30–16:30 Song Sun (Stony Brook)
  "Kähler-Einstein metrics and Gromov-Hausdorff limits"
  I will talk about the algebraic structure of Gromov-Hausdorff limits of Kähler-Einstein manifolds, and its relation with the existence theory of Kähler-Einstein metrics on Fano manifolds, as well as the theory of moduli spaces.

July 8th (Tuesday)

- 9:30–10:30 Dietmar Salamon (ETH)
  "A differential geometric approach to GIT"
  In this lecture I will explain the moment-weight inequality, and its role in the proof of the Hilbert-Mumford numerical criterion for stability. The setting is Hamiltonian group actions on closed Kähler manifolds. The moment-weight inequality relates the Mumford numerical invariants to the norm of the moment map on the complexified group orbit. Key ingredients in the proof are the negative gradient flow of the moment map squared and the Kemp-Ness function. In the analogy between finite-dimensional GIT and CSCK metrics the Mumford numerical invariants correspond to the Futaki invariants and the Kempf-Ness function corresponds to the Mabuchi functional. This is joint work with Valentina Georgoulas and Joel Robbin, based on conversations with Xiuxiong Chen, Song Sun, and Sean Paul.

- 11:00–12:00 Jian Song (Rutgers)

  "Riemannian geometry of Kähler-Einstein currents"

  We study Riemannian geometry of canonical Kähler-Einstein currents on projective Calabi-Yau varieties and canonical models of general type with crepant resolutions. We prove that the metric completion of the regular part of such a canonical current is a compact metric length space homeomorphic to the original projective variety with well defined tangent cones. A number of applications are given for degenerations of Calabi-Yau manifolds and the Kähler-Ricci flow on smooth minimal models of general type.

- 14:00-15:00 Shouhei Honda (Kyushu)

  "Cheeger constant, p-Laplacian, and Gromov-Hausdorff convergence"

  In this talk we introduce a Rellich type compactness with respect to the Gromov-Hausdorff topology. This compactness has several applications. For example we can give an alternative simplified proof of Fukaya's conjecture on the behavior of the eigenvalues of the Laplacian with respect to the GH-topology which was proved by Cheeger-Colding, In this talk we particularly discuss a new relationship between Cheeger constant, p-Laplacian, and Gromov-Hausdorff convergence obtained by this compactness.

- 15:30–16:30 Takao Yamaguchi (Kyoto)

  "Lipschitz homotpy structure of Alexandrov spaces"

  Alexandrov spaces with curvature bounded below typically appear as Gromov-Hausdorff limits of Riemannian manifolds with a lower curvature bound, and the study of such spaces have been an important subject in Riemannian geometry. It sometimes brings us significant information of global structure of manifolds. Local homeomorphism structure of Alexandrov spaces is known as Perelman's topological stability theorem. However Lipschitz structure is not known at this stage, although Perelman claimed it long time ago. In this lecture, I will talk about Lipshitz homotopy structure based on recent joint works with Ayato Mitsuishi. We make use of gradient curves for distance functions to establish Lipschitz homotopy version of critical point theory. Several applications will also be presented.

July 9th (Wednesday)

- 9:30–10:30 Olivier Biquard (ENS)
  "Einstein metrics, complex singularities, and the AH problem"
  I will discuss the obstruction I found some time ago for desingularizing Einstein 4-orbifolds, which leads to a wall crossing phenomenon in the Dirichlet problem for Asymptotically Hyperbolic Einstein metrics.

- 11:00–12:00 Naichung Conan Leung (CUHK)
  "Gradient flow trees from Witten deformation"
  We studied Witten deformation of Laplacian operators twisted by Morse functions. In the semi-classical limit, we show that the A_∞ structure on small eigenforms converges to the one in Morse category defined by counting gradient flow trees. This is a joint work with K.L. Chan and Z.M. Ma. This research work is supported by RGC grant CUHK 2130314.

July 10th (Thursday)

- 9:30–10:30 Claudio Arezzo (ICTP)
  "Constant Scalar Curvature Kähler metrics on Resolutions of singularities"
  I will describe a new gluing result for Kähler constant scalar curvature metrics obtained in collaboration with R. Lena and L. Mazzieri. This construction generalised to Kcsc metrics of any sign the celebrated Kummer construction for Calabi-Yau manifolds discovered by LeBrun and Singer.

- 11:00–12:00 Sean Paul (Wisconsin-Madison)
  "Introduction to semistable pairs and applications"
  The speaker will give an account of his recent work on the algebraic geometry that lies behind Mabuchi's K-energy map.

- 13:30–14:30 Ngaiming Mok (Hong Kong)
  "Geometric structures and substructures on uniruled projective manifolds"
  With J.-M. Hwang the speaker has developed a geometric theory of uniruled projective manifolds X modeled on varieties of minimal rational tangents $\mathcal{C}_x(X) \subset \mathbb{P}T_x(X)$, alias VMRTs. Generalizing works of Hwang-Mok, Hong-Mok

considered pairs $(X_0; X)$ of uniruled projective manifolds, and established a non-equidimensional Cartan-Fubini Extension Principle (2010) in terms of a certain non-degeneracy condition on the second fundamental form for a pair $(\mathcal{B} \subset \mathcal{A})$ consisting of a VMRT $\mathcal{A}$ and a linear section $\mathcal{B}$ of $\mathcal{A}$. The latter has led to the characterization of standard embeddings $i : G_0/P_0 \hookrightarrow G/P$ between rational homogeneous manifolds of Picard number 1 by Hong-Mok (2010) in the long-root and non-linear cases and by Hong-Park (2011) in the short-root cases and in the cases of linear subspaces with identifiable exceptions. The argument therein involving parallel transport of VMRTs has also been applied by Hong-Mok (2013) to establish homological rigidity for certain smooth Schubert cycles.

Recently in a joint work with Y. Zhang we have established a stronger rigidity phenomenon for sub-VMRT structures, where in place of a germ of mapping $f : (X_0; 0) \to (X; 0)$ we consider a germ of submanifold $(S; 0) \subset (X; 0)$ for a uniruled projective manifold X equipped with a minimal rational component $\mathcal{K}$. Defining a sub-VMRT structure by taking intersections $\mathcal{C}_x(X) \cap \mathbb{P}T_x(S)$ we have obtained sufficient conditions for S to extend to a rationally saturated projective subvariety $Z \subset X$. In the rational homogeneous case the method yields a strengthening of the results of Hong-Mok and Hong-Park. For instance, if a germ of submanifold $(S; 0) \subset (X; 0)$ inherits by intersecting VMRTs with projectivized tangent subspaces a Grassmann structure of rank ≥ 2, then S in fact extends to a sub-Grassmannian in its standard embedding.

- 15:00–16:00 Kenji Fukaya (SCGP)
  "Sheaf of perturbation data in de Rham version of virtual fundamental chain"

- 16:30–17:30 Gang Tian (Beijing/Princeton)
  "An extension of Matsushima Theorem"
  In this talk, I will give a version of Matsushima Theorem for normal varieties. I will explain its proof following the original approach of Matsushima and its generalization in my 2012 paper.

July 11st (Friday)

- 9:30–10:30 Yuji Sano (Kumamoto)
  "On the extremal vector fields on smooth toric Fano manifolds"
  On a compact Kähler manifold, Futaki and Mabuchi showed that the complex gradient vector field of the projection of the scalar curvature to the space of the normalized Hamiltonian functions of holomorphic vector fields is independent of the choice of Kähler metrics. Such vector field is called the extremal vector field. In this talk, I will explain about a computation on the extremal vector fields on smooth toric Fano manifolds.

- 11:00–12:00 Kaoru Ono (Kyoto)
  "Floer complex and covering spaces"
  Arnold's conjecture for fixed points of Hamiltonian diffeomorphisms has stimulated the development of symplectic geometry. It was a motivation of Floer to initiate what is now called Floer cohomology. Although the Betti number version of the conjecture was settled, the original form of the conjecture is still open. In particular, whether the non-triviality of the fundamental group implies some additional lower bound or not. Recently, there are some progress, though not so much. I will explain an approach based on a joint work with Andrei Pajitnov. If time allows, I may touch another direction of study.