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Lipschitz stability in an inverse hyperbolic problem with impulsive forces

by

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Abstract

Let $u = u(q)$ satisfy a hyperbolic equation with impulsive input:

$$\partial_t^2 u(x,t) - \triangle u(x,t) + q(x)u(x,t) = \delta(x_1)\delta'(t)$$

and let $u|_{t<0} = 0$. Then we consider an inverse problem of determining $q(x)$, $x \in \Omega$ from data $u(q)|_{S_T}$ and $(\partial u(q)/\partial\nu)\,|_{S_T}$. Here $\Omega \subset \{(x_1,\dots,x_n) \in \mathbb{R}^n | x_1 > 0\}$, $n \geq 2$, is a bounded domain, $S_T = \{(x,t);\ x \in \partial\Omega,\ x_1 < t < T + x_1\}$, $\nu = \nu(x)$ is the unit outward normal vector to $\partial\Omega$ at $x \in \partial\Omega$, and $T > 0$. For suitable $T > 0$, we prove an estimate:

$$\|q_1 - q_2\|_{L^2(\Omega)} \leq C\left\{ \|u(q_1) - u(q_2)\|_{H^1(S_T)} + \left\| \frac{\partial u(q_1)}{\partial\nu} - \frac{\partial u(q_2)}{\partial\nu} \right\|_{L^2(S_T)} \right\},$$

provided that q_1 satisfies a boundedness condition and q_2 satisfies a smallness condition in the Sobolev norm of order $n+2$.

1 Introduction and main results

We consider an inverse problem of determining a coefficient in a hyperbolic equation by an impulsive source located outside the domain where a coefficient

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is unknown. Let $x = (x_1, ..., x_n)$, $n \geq 2$ and $t \in \mathbb{R}$. Let $u(x,t)$ solve the Cauchy problem in $(x,t) \in \mathbb{R}^{n+1}$:

$$\partial_t^2 u(x,t) - \triangle u(x,t) + q(x)u(x,t) = \delta(x_1)\delta'(t), \quad u|_{t<0} = 0, \tag{1.1}$$

where δ and δ' are the Dirac delta function and the t-derivative:

$$\langle \delta(x_1), \psi \rangle = \psi\left(0, x_2, \dots, x_n, t\right),$$

and

$$\langle \delta'(t), \psi \rangle = -\partial_t \psi(x, 0), \ \forall \psi \in C_0^\infty\left(\mathbb{R}^{n+1}\right).$$

As for the regularity of the solution, see Proposition 2.2 in Section 2.

Let $\mathbb{R}^n_+ = \{(x_1, \dots, x_n) \in \mathbb{R}^n | x_1 > 0\}$ and let $\Omega \subset \mathbb{R}^n_+$ be a bounded domain with C^1-piecewise smooth boundary $\partial\Omega$. Furthermore let $T > 0$ be suitably given. Set

$$G_T = \{(x,t);\ x \in \Omega,\ x_1 < t < T + x_1\}, \tag{1.2}$$

$$\Sigma_0 = \{(x,t);\ x \in \Omega,\ t = x_1 + 0\},$$

$$\Sigma_T = \{(x,t);\ x \in \Omega,\ t = T + x_1\},$$

$$S_T = \{(x,t);\ x \in \partial\Omega,\ x_1 < t < T + x_1\}. \tag{1.3}$$

We consider:

Inverse problem. Let Cauchy data of the solution u to (1.1) be given on S_T:

$$u(x,t) = f(x,t), \quad \frac{\partial u}{\partial \nu}(x,t) = g(x,t), \quad (x,t) \in S_T, \tag{1.4}$$

where $\nu = \nu(x)$ is the unit outward normal vector to $\partial\Omega$ at $x \in \partial\Omega$. Then determine $q(x)$, $x \in \Omega$ from given data (1.4).

If we can assume the positivity condition $u(\cdot, 0) > 0$ on $\overline{\Omega}$, then the method on the basis of a Carleman estimate which was discussed first in Bukhgeim and Klibanov [2], implies the uniqueness. As for the stability, see Imanuvilov and Yamamoto [5, 6], Khaĭdarov [10], Yamamoto [22], and we refer also to Isakov [7, 8, 9], Klibanov [11], Klibanov and Timonov [12]. In (1.1) we take an impulsive input $\delta(x_1)\delta'(t)$ and the initial values can be zero. The impulsive input is acceptable from the practical viewpoint.

This paper aims at the stability in this inverse hyperbolic problem without positivity of $u(\cdot,0)$ by a single measurement, which is a longstanding open problem. Theorem 1.1 stated below is a partial answer to the open problem.

In order to state the main result, we introduce notations. Let $r =$(diam $\Omega)/2$. Assume that

$$\begin{aligned} &\Omega \subseteq B\left(x^0, r\right) = \left\{x \in \mathbb{R}^n;\ \left|x - x^0\right| < r\right\} \\ &\text{where } x^0 = \left(x_1^0, 0, \ldots, 0\right) \in \mathbb{R}_+^n \text{ and } x_1^0 > r > 0. \end{aligned} \tag{1.5}$$

Set

$$K = K\left(x^0, T, r\right) = \left\{(x,t);\ |x_1| < t < \left(T + x_1^0 + 2r\right) - \left|x - x^0\right|\right\}.$$

Noting that $x_1 > 0$ and $T + x_1 \leq T + x_1^0 + r \leq \left(T + x_1^0 + 2r\right) - \left|x - x^0\right|$ for $x \in \Omega$, we see that $G_T \subseteq K$. Denote by

$$P = P\left(x^0, T, r\right) = \left\{x \in \mathbb{R}^n;\ |x_1| < \left(T + x_1^0 + 2r\right) - \left|x - x^0\right|\right\}$$

the projection of K on the space $\mathbb{R}^n$. Throughout this paper, $H^1(S_T)$, $H^{n+2}(P)$, etc. denote usual Sobolev spaces (e.g. Adams [1]), and $[\alpha]$ denotes the greatest integer not exceeding α. We set

$$\mathcal{U}(Q) = \left\{q \in H^{n+2}(P) \,\middle|\, \|q\|_{H^{n+2}(P)} \leq Q\right\} \tag{1.6}$$

for any fixed $Q > 0$. Furthermore, we take a constant β such that

$$0 < \beta < 1 \text{ and } 0 < \beta\left(r\beta + x_1^0 + 2r\right)^2 < (x_1^0 - r)^2. \tag{1.7}$$

Now we state the main result.

Theorem 1.1. *Assume that Ω satisfies (1.5). Let $M > 0$ and*

$$T > 2r + \frac{4(x_1^0 + 2r)}{\beta} \tag{1.8}$$

where β satisfies (1.7). Suppose that $q_1 \in \mathcal{U}(M)$ and $q_2 \in \mathcal{U}(\varepsilon)$ for $\varepsilon > 0$. Furthermore, let u_k be the solution to (1.1) with $q = q_k$, f_k and g_k Cauchy data in (1.4) for $u = u_k$, $k = 1,2$. Then there exist constants $\varepsilon_0 = \varepsilon_0\left(\Omega, T, x^0, r, M, \beta\right) > 0$ and $C = C\left(\Omega, T, x^0, r, M, \beta, \varepsilon_0\right) > 0$ such that for any $0 < \varepsilon \le \varepsilon_0$ the following estimate

$$\|q_1 - q_2\|_{L^2(\Omega)} \le C\left(\|f_1 - f_2\|_{H^1(S_T)} + \|g_1 - g_2\|_{L^2(S_T)}\right) \tag{1.9}$$

holds for any $q_1 \in \mathcal{U}(M)$, $q_2 \in \mathcal{U}(\varepsilon)$.

Estimate (1.8) establishes the Lipschitz stability which is the best possible for the inverse problem, but we need the smallness for either of q_1 and q_2. We can interpret the setting of the theorem as the determination of not necessarily small q_1 around fixed but small q_2 (i.e., $q_2 \in \mathcal{U}(\varepsilon)$).

If we can be allowed to repeat infinitely many measurements, then the Dirichlet to Neumann map can guarantee the uniqueness and the stability with the zero initial condition (e.g., Sun [21]).

The above referred results by a Carleman estimate or the Dirichlet to Neumann map, hold without smallness assumptions of unknown coefficients or the spatial domain Ω under consideration. However, the Dirichlet to Neumann map requires the infinitely many repeats of the measurements, which is not realistic. On the other hand, the positivity of the initial displacement which an approach by Carleman estimate needs, may be difficult to be realized in practise even though a single measurement can guarantee the uniqueness and the stability in the inverse problem.

In the case where the spatial dimension is greater than 1, it is a hard open problem whether in the inverse problem for (1.1), one can establish the uniqueness without any smallness conditions on the coefficients or Ω. In Romanov and Yamamoto [18], if both $\|q_1\|_{H^{n+2}(P)}$ and $\|q_2\|_{H^{n+2}(P)}$ are sufficiently small, then with suitable T, we can prove the Lipschitz stability for $\|q_1 - q_2\|_{L^2(\Omega)}$ by means of the boundary data. As related results, see Glushkov [3], Glushkov and

Romanov [4], Romanov [14, 15, 16, 17], Romanov and Yamamoto [18, 19, 20]. In Li [13], assuming that $q_2 \equiv 0$, the stability is proved, which means an L^2-size estimate of a coefficient by the boundary measurement.

Our proof is inspired by the argument in §4.1 in [17] and [18], but we will use an inequality of novel Carleman type.

2 Proof of Theorem 1.1

First we show a new Carleman inequality estimating also the solution on the characteristics, which is an independent interest. For $T > 0$, $x_1^0 > 0$ and $\beta \in (0,1)$, we define a function $\varphi = \varphi(x,t)$ by

$$\varphi(x,t) = \frac{1}{4}|x|^2 - \frac{1}{8}\beta\left(t - x_1^0 - \frac{T}{2}\right)^2. \tag{2.1}$$

Furthermore, we set

$$\partial_t = \frac{\partial}{\partial t}, \quad \partial_j = \frac{\partial}{\partial x_j}, \quad 1 \le j \le n, \quad \nabla_x = (\partial_1, \ldots, \partial_n),$$

$$\nabla_{x,t} = (\partial_1, \ldots, \partial_n, \partial_t), \quad \nabla_{x'} = (\partial_2, \ldots, \partial_n), \quad \Box y = \partial_t^2 y - \triangle y.$$

Proposition 2.1 ([13])**.** *Let $v \in H^2(G_T)$. Assume (1.5), (1.7) and (1.8). Then there exists a constant $\vartheta > 0$ such that for $T \in \left(2r + 4\left(x_1^0 + 2r\right)/\beta,\right.$ $\left. 2r + 4\left(x_1^0 + 2r\right)/\beta + \vartheta\right)$ there exist $s_0 > 0$ and $C_1 = C_1(s_0, T, x^0, r, \beta) > 0$ such that*

$$\begin{aligned}
&\int_{G_T} \left(s\left|\nabla_{x,t} v\right|^2 + s^3 v^2\right) \mathrm{e}^{2s\varphi} \mathrm{d}x\mathrm{d}t \\
&+ \int_{\Sigma_0 \cup \Sigma_T} \left(s\left(\partial_t v + \partial_1 v\right)^2 + s\left|\nabla_{x'} v\right|^2 + s^3 v^2\right) \mathrm{e}^{2s\varphi} \mathrm{d}x \\
&\le C_1 \left\{ \int_{G_T} (\Box v)^2 \mathrm{e}^{2s\varphi} \mathrm{d}x\mathrm{d}t + \int_{S_T} \left(s\left|\nabla_{x,t} v\right|^2 + s^3 v^2\right) \mathrm{e}^{2s\varphi} \mathrm{d}\sigma\mathrm{d}t \right\}
\end{aligned} \tag{2.2}$$

for all $s \ge s_0$.

An estimate on $\Sigma_0 \cup \Sigma_T$ is given by Romanov [17] (Lemma 4.1.4), but in [17] any weight function with large parameter s, is not considered. On the other hand, Proposition 2.1 is attached by a weight function with a large parameter, which is an inequality of Carleman's type. In Li [13], the proof of Proposition 2.1 is given, and for the completeness of statement, we will prove Proposition 2.1 in the appendix.

In [17, 18], the following proposition is proved.

Proposition 2.2 ([17, 18])**.** *. Let $q \in \mathcal{U}(Q)$. Then the solution to (1.1) can be represented in the form*

$$u(x,t) = \frac{1}{2}\delta\left(t - |x_1|\right) + \widehat{u}(x,t)\theta_0\left(t - |x_1|\right) \tag{2.3}$$

where $\widehat{u} \in H^m(K)$, $m = \left[\frac{n+1}{2}\right]+1$, $\theta_0(t)$ is the Heaviside step function: $\theta_0(t) = 1$ if $t \geq 0$ and $\theta_0(t) = 0$ if $t < 0$. Moreover

$$\widehat{u}(x, |x_1| + 0) = -\frac{1}{4}(sign\ x_1)\int_0^{x_1} q(\xi, x')\mathrm{d}\xi, \quad x \in P \tag{2.4}$$

with $x' = (x_2, \ldots, x_n)$, and there exists a constant $C_2 = C_2(T, x^0, r, Q) > 0$ such that

$$|\widehat{u}(x,t)| \leq C_2 Q, \quad (x,t) \in K. \tag{2.5}$$

The constant C_2 is a non-decreasing function of parameters T, r, Q.

Remark 2.1. The representation (2.3) means that the regular part of the solution $u(x,t)$ coincides with $\widehat{u}(x,t)$ for $(x,t) \in K$. Moreover $u \in H^1(S_T)$ and $\partial u/\partial \nu \in L^2(S_T)$ by the trace theorem (e. g., [1]), because $\partial\Omega$ is piecewise C^1 smooth and $u \in H^m(K)$ with $m \geq 2$.

Now we prove Theorem 1.1.

Proof of Theorem 1.1. For any $T > 0$ satisfying (1.8), we set

$$\widetilde{T} = \min\left\{T, 2r + \frac{4\left(x_1^0 + 2r\right)}{\beta} + \frac{\vartheta}{2}\right\}, \tag{2.6}$$

where ϑ is given by Proposition 2.1. Therefore, estimate (2.2) holds in $G_{\widetilde{T}}$.

We set

$$y = u_1 - u_2, \quad p = q_1 - q_2. \tag{2.7}$$

Then by

$$\Box u_k(x,t) + q_k(x)u_k(x,t) = 0, \quad (x,t) \in G_{\widetilde{T}},$$

we have

$$\Box y(x,t) + q_1(x)y(x,t) + p(x)u_2(x,t) = 0, \quad (x,t) \in G_{\widetilde{T}}. \tag{2.8}$$

By $q_1 \in \mathcal{U}(M)$ and the embedding theorem, we see that $q_1 \in C(P)$ and there exists a constant $C_0 = C_0(T, x^0, r, \Omega) > 0$ such that

$$\|q_1\|_{C(P)} \leq C_0 \|q_1\|_{H^{n+2}(P)} \leq C_0 M. \tag{2.9}$$

By $q_2 \in \mathcal{U}(\varepsilon)$ and (2.5) in Proposition 2.2, we have

$$|u_2(x,t)| \leq C_2 \varepsilon, \quad (x,t) \in G_{\widetilde{T}}, \tag{2.10}$$

It follows from (2.8), (2.9) and (2.10) that

$$(\Box y(x,t))^2 \leq 2C_0^2 M^2 y^2(x,t) + 2C_2^2 \varepsilon^2 p^2(x), \quad (x,t) \in G_{\widetilde{T}}.$$

Then, by Proposition 2.1, there exists $s_0 > 0$ such that

$$\begin{aligned}
&\int_{G_{\widetilde{T}}} \left(s\,|\nabla_{x,t} y|^2 + s^3 y^2 \right) \mathrm{e}^{2s\varphi} \mathrm{d}x\mathrm{d}t \\
&+ \int_{\Sigma_0 \cup \Sigma_{\widetilde{T}}} \left(s\,(\partial_t y + \partial_1 y)^2 + s\,|\nabla_{x'} y|^2 + s^3 y^2 \right) \mathrm{e}^{2s\varphi} \mathrm{d}x \\
&\leq C_1 \left(\int_{G_{\widetilde{T}}} (\Box y)^2 \, \mathrm{e}^{2s\varphi} \mathrm{d}x\mathrm{d}t + \int_{S_{\widetilde{T}}} \left(s\,|\nabla_{x,t} y|^2 + s^3 y^2 \right) \mathrm{e}^{2s\varphi} \mathrm{d}\sigma\mathrm{d}t \right) \\
&\leq C_1 \left(2C_0^2 M^2 \int_{G_{\widetilde{T}}} y^2 \mathrm{e}^{2s\varphi} \mathrm{d}x\mathrm{d}t + 2C_2^2 \varepsilon^2 \int_{G_{\widetilde{T}}} p^2 \mathrm{e}^{2s\varphi} \mathrm{d}x\mathrm{d}t \right. \\
&\left. + \int_{S_{\widetilde{T}}} \left(s\,|\nabla_{x,t} y|^2 + s^3 y^2 \right) \mathrm{e}^{2s\varphi} \mathrm{d}\sigma\mathrm{d}t \right)
\end{aligned} \tag{2.11}$$

for all $s > s_0$, where $\varphi = \varphi(x,t)$ is defined by (2.1).

By (2.4) in Proposition 2.2, we have

$$\partial_t y + \partial_1 y = -\frac{1}{4} p(x), \quad (x,t) \in \Sigma_0. \tag{2.12}$$

It follows from (2.11) and (2.12) that

$$
\begin{aligned}
&\int_{G_{\widetilde{T}}} \left(s\left|\nabla_{x,t} y\right|^2 + \left(s^3 - 2C_1 C_0^2 M^2\right) y^2 \right) \mathrm{e}^{2s\varphi} \mathrm{d}x\mathrm{d}t \\
&+ \frac{s}{16} \int_\Omega p^2(x) \mathrm{e}^{2s\varphi(x,x_1)} \mathrm{d}x \\
&\le 2C_1 C_2^2 \varepsilon^2 \int_{G_{\widetilde{T}}} p^2 \mathrm{e}^{2s\varphi} \mathrm{d}x\mathrm{d}t + C_1 \int_{S_{\widetilde{T}}} \left(s\left|\nabla_{x,t} y\right|^2 + s^3 y^2 \right) \mathrm{e}^{2s\varphi} \mathrm{d}\sigma\mathrm{d}t
\end{aligned}
\tag{2.13}
$$

for all $s > s_0$. We take $s_1 > \max\left\{ s_0, \sqrt[3]{2C_1 C_0^2 M^2} \right\}$ and fix it. Then using (2.13) and noting $G_{\widetilde{T}} \subseteq \Omega \times \left(0, \widetilde{T} + x_1^0 + r\right)$, we have

$$
\begin{aligned}
\frac{1}{16} s_1 \mathrm{e}^{2s_1\Phi_1} \int_\Omega p^2 \mathrm{d}x \le C_1 \mathrm{e}^{2s_1\Phi_2} \Bigg\{ & 2C_2^2 \varepsilon^2 \left(\widetilde{T} + x_1^0 + r \right) \int_\Omega p^2 \mathrm{d}x \\
& + \int_{S_{\widetilde{T}}} \left(s_1 \left|\nabla_{x,t} y\right|^2 + s_1^3 y^2 \right) \mathrm{d}\sigma\mathrm{d}t \Bigg\},
\end{aligned}
\tag{2.14}
$$

where $\Phi_1 = \inf_{(x,t)\in\overline{G_{\widetilde{T}}}} \varphi(x,t)$ and $\Phi_2 = \sup_{(x,t)\in\overline{G_{\widetilde{T}}}} \varphi(x,t)$. We choose $\varepsilon_0 > 0$ such that

$$
\frac{1}{16} s_1 \mathrm{e}^{2s_1\Phi_1} > 2C_1 C_2^2 \varepsilon_0^2 \left(\widetilde{T} + x_1^0 + r \right) \mathrm{e}^{2s_1\Phi_2}
$$

and fix ε_0. Then it follows from (2.14) that, for any $\varepsilon \in (0, \varepsilon_0]$, there exists a constant $C_* = C_*(T,\, x_1^0,\, r,\, \Omega,\, M,\, \beta,\, \varepsilon_0)$ such that

$$
\int_\Omega p^2 \mathrm{d}x \le C_* \int_{S_{\widetilde{T}}} \left(\left|\nabla_{x,t} y\right|^2 + y^2 \right) \mathrm{d}\sigma\mathrm{d}t \tag{2.15}
$$

for any $q_1 \in \mathcal{U}(M)$, $q_2 \in \mathcal{U}(\varepsilon)$. By (1.3), (1.4), (2.6), (2.7), and (2.15), we obtain (1.9). The proof of Theorem 1.1 is completed. □

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Appendix. Proof of Proposition 2.1

The proof is inspired by Lemma 4.1.4 in [17], but we have to treat the weight function carefully. First of all, we note that the following inequalities hold:

$$0 < x_1^0 - r \le x_1 \le x_1^0 + r, \quad 0 < |x|^2 \le \left(r + x_1^0\right)^2, \qquad x \in \overline{\Omega}, \tag{A.1}$$

and
$$-r - \frac{T}{2} \le t - x_1^0 - \frac{T}{2} \le \frac{T}{2} + r, \qquad (x,t) \in \overline{G_T}. \tag{A.2}$$

In fact, the first inequality in (A.1) follows from (1.5). The second inequality in (A.1) can be proved as follows:

$$|x|^2 = |x - x^0|^2 + x_1^2 - (x_1 - x_1^0)^2 \le r^2 + 2x_1 x_1^0 - (x_1^0)^2 \le r^2 + 2x_1^0(x_1^0 + r) - (x_1^0)^2.$$

(A.2) can be proved by (1.2) and (A.1).

By (1.7), there exists a constant ϑ such that

$$0 < \frac{1}{16}\beta^3 \left(4r + \frac{4\left(x_1^0 + 2r\right)}{\beta} + \vartheta\right)^2 < (x_1^0 - r)^2. \tag{A.3}$$

By (1.8), we can assume that $T \in \left(2r + 4\left(x_1^0 + 2r\right)/\beta, 2r + 4\left(x_1^0 + 2r\right)/\beta + \vartheta\right)$. Then we have

$$4r + \frac{4\left(x_1^0 + 2r\right)}{\beta} < T + 2r < 4r + \frac{4\left(x_1^0 + 2r\right)}{\beta} + \vartheta. \tag{A.4}$$

It follows from (A.3) and (A.4) that

$$(x_1^0 - r)^2 > \frac{1}{16}\beta^3 \left(T + 2r\right)^2.$$

Therefore we can take a constant $\rho > 0$ such that

$$0 < 2\beta < \rho < \min\left\{2, \frac{64\left(x_1^0 - r\right)^2}{\beta^2\left(T + 2r\right)^2} - 2\beta\right\}. \tag{A.5}$$

Furthermore, by (1.8), we can obtain

$$\frac{\beta^2}{16}\left(\frac{T}{2} - r\right)^2 > \frac{1}{4}\left(2r + x_1^0\right)^2 \qquad \text{and} \qquad \frac{\beta T}{4} - \frac{\beta r}{2} - x_1^0 > 2r. \tag{A.6}$$

Let $s > 0$, $w = \mathrm{e}^{s\varphi} v$ and $Lw = \mathrm{e}^{s\varphi} \Box \left(\mathrm{e}^{-s\varphi} w\right)$. Then we obtain that

$$\begin{aligned} Lw = &\left\{ \Box w + s^2 \left((\partial_t \varphi)^2 - |\nabla_x \varphi|^2 \right) w + \tfrac{1}{4} s\rho w \right\} \\ &+ s \left\{ \left(-\Box\varphi - \tfrac{1}{4}\rho \right) w - 2 (\partial_t \varphi)(\partial_t w) + 2 (\nabla_x \varphi \cdot \nabla_x w) \right\} \\ = &\left(\Box w + s^2 dw + \tfrac{1}{4} s\rho w \right) + s \left(cw + b (\partial_t w) + a \cdot \nabla_x w \right) \end{aligned}$$

where $a = 2\nabla_x \varphi = x$, $b = -2(\partial_t \varphi) = \beta \left(t - x_1^0 - T/2\right)/2$, $c = -\Box\varphi - \rho/4 = \beta/4 + n/2 - \rho/4$ and $d = (\partial_t \varphi)^2 - |\nabla_x \varphi|^2 = \beta^2 \left(t - x_1^0 - T/2\right)^2/16 - |x|^2/4$. We note that c is a constant. Furthermore, by $\rho < 2$, we have

$$c > \frac{\beta}{4} + \frac{n}{2} - \frac{1}{2} \geq \frac{\beta}{4}.$$

Using the inequality: $(\alpha + \gamma)^2 \geq 2\alpha\gamma$, we have

$$(Lw)^2 \geq 2s \left(\Box w + s^2 dw + \frac{1}{4} s\rho w \right) \left(cw + b (\partial_t w) + a \cdot \nabla_x w \right). \tag{A.7}$$

Noting that

$$a = x, \quad b = \frac{1}{2}\beta \left(t - x_1^0 - \frac{T}{2} \right), \quad \text{and} \quad c = \frac{\beta}{4} + \frac{n}{2} - \frac{\rho}{4}, \tag{A.8}$$

we can verify that

$$2 (\Box w) \left(cw + b (\partial_t w) + a \cdot \nabla_x w \right) = \partial_t P + \nabla_x \cdot Q + R$$

where

$$P = b \left((\partial_t w)^2 + |\nabla_x w|^2 \right) + 2 (\partial_t w) \left(a \cdot \nabla_x w + cw \right), \tag{A.9}$$

$$Q = \left(|\nabla_x w|^2 - (\partial_t w)^2 \right) a - 2 \left(a \cdot \nabla_x w + b (\partial_t w) + cw \right) (\nabla_x w), \tag{A.10}$$

$$R = \frac{1}{2} (\rho - 2\beta) (\partial_t w)^2 + \frac{1}{2} (4 - \rho) |\nabla_x w|^2. \tag{A.11}$$

Therefore,

$$\begin{aligned} &2 \int_{G_T} (\Box w) \left(cw + b (\partial_t w) + a \cdot \nabla_x w \right) \mathrm{d}x\mathrm{d}t \\ &= \int_{\Sigma_T} (P - Q_1) \, \mathrm{d}x + \int_{\Sigma_0} (Q_1 - P) \, \mathrm{d}x + \int_{S_T} Q \cdot \nu \mathrm{d}\sigma\mathrm{d}t + \int_{G_T} R \mathrm{d}x\mathrm{d}t. \end{aligned} \tag{A.12}$$

By (A.9) and (A.10), we can obtain that

$$\begin{aligned}P-Q_1 &= (b+a_1)\,(\partial_t w+\partial_1 w)^2+(b-a_1)\,|\nabla_{x'}w|^2\\ &\quad+2\,(\partial_t w+\partial_1 w)\,(a'\cdot\nabla_{x'}w+cw)\,,\end{aligned}\tag{A.13}$$

where $a'=(a_2,\dots,a_n)$. Then by $x_1^0>0$ and the inequality: $|(a'\cdot\nabla_{x'}w)|\le |a'|\,|\nabla_{x'}w|$, we have

$$\begin{aligned}P-Q_1 &\ge (b+a_1)\,(\partial_t w+\partial_1 w)^2+(b-a_1)\,|\nabla_{x'}w|^2\\ &\quad -2x_1^0\,(\partial_t w+\partial_1 w)^2-\frac{1}{2x_1^0}\,(a'\cdot\nabla_{x'}w)^2+2c\,(\partial_t w+\partial_1 w)\,w\\ &\ge \left(b+a_1-2x_1^0\right)(\partial_t w+\partial_1 w)^2+\left(b-a_1-\frac{1}{2x_1^0}\,|a'|^2\right)|\nabla_{x'}w|^2\\ &\quad +2c\,(\partial_t w+\partial_1 w)\,w.\end{aligned}$$

By (A.1) and (A.6), we have

$$\begin{aligned}& b+a_1-2x_1^0=\frac{1}{2}\beta\left(x_1-x_1^0+\frac{T}{2}\right)+x_1-2x_1^0\\ &\ge \frac{1}{2}\beta\left(\frac{T}{2}-r\right)+x_1^0-r-2x_1^0=\frac{1}{4}\beta T-\frac{1}{2}\beta r-x_1^0-r\\ &> r, \qquad (x,t)\in\Sigma_T.\end{aligned}$$

By (1.5), (A.1) and (A.6), we have

$$\begin{aligned}& b-a_1-\frac{1}{2x_1^0}\,|a'|^2=\frac{1}{2}\beta\left(x_1-x_1^0+\frac{T}{2}\right)-x_1-\frac{1}{2x_1^0}\sum_{j=2}^{n}{x_j}^2\\ &\ge \frac{1}{2}\beta\left(\frac{T}{2}-r\right)-x_1^0-r-\frac{r^2}{2r}=\frac{1}{4}\beta T-\frac{1}{2}\beta r-x_1^0-\frac{3}{2}r\\ &> \frac{r}{2}, \qquad (x,t)\in\Sigma_T.\end{aligned}$$

Therefore,

$$P-Q_1\ge r\,(\partial_t w+\partial_1 w)^2+\frac{1}{2}r\,|\nabla_{x'}w|^2+\partial_1\left(cw^2\big|_{\Sigma_T}\right), \qquad (x,t)\in\Sigma_T. \tag{A.14}$$

Similarly, by (A.13), we have

$$\begin{aligned}Q_1-P &= (-b-a_1)\,(\partial_t w+\partial_1 w)^2+(a_1-b)\,|\nabla_{x'}w|^2\\ &\quad -2\,(\partial_t w+\partial_1 w)\,(a'\cdot\nabla_{x'}w+cw)\\ &\ge \left(-b-a_1-\frac{r}{2}\right)(\partial_t w+\partial_1 w)^2+\left(a_1-b-\frac{2}{r}\,|a'|^2\right)|\nabla_{x'}w|^2\\ &\quad -2c\,(\partial_t w+\partial_1 w)\,w.\end{aligned}$$

By (A.1) and (A.6), we have

$$\begin{aligned}
&-b-a_1-\frac{r}{2}=\frac{1}{2}\beta\left(x_1^0-x_1+\frac{T}{2}\right)-x_1-\frac{r}{2}\\
&\geq\frac{1}{2}\beta\left(\frac{T}{2}-r\right)-x_1^0-r-\frac{r}{2}=\frac{1}{4}\beta T-\frac{1}{2}\beta r-x_1^0-\frac{3}{2}r\\
&>\frac{r}{2},\qquad (x,t)\in\Sigma_0.
\end{aligned}$$

By (1.5), (A.1) and (A.6), we have

$$\begin{aligned}
&a_1-b-\frac{2}{r}\left|a'\right|^2=x_1-\frac{1}{2}\beta\left(x_1-x_1^0-\frac{T}{2}\right)-\frac{2}{r}\sum_{j=2}^{n}{x_j}^2\\
&\geq x_1^0-r-\frac{1}{2}\beta\left(x_1^0+r-x_1^0-\frac{T}{2}\right)-\frac{2}{r}r^2=\frac{1}{4}\beta T-\frac{1}{2}\beta r+x_1^0-3r\\
&\geq\left(2r+x_1^0\right)+x_1^0-3r=2x_1^0-r>r,\qquad (x,t)\in\Sigma_0.
\end{aligned}$$

Therefore,

$$Q_1-P\geq\frac{1}{2}r\left(\partial_t w+\partial_1 w\right)^2+r\left|\nabla_{x'}w\right|^2-\partial_1\left(cw^2\big|_{\Sigma_0}\right),\qquad (x,t)\in\Sigma_0.\quad\text{(A.15)}$$

By (A.10), we have

$$Q\cdot\nu=\left(\left|\nabla_x w\right|^2-\left(\partial_t w\right)^2\right)(a\cdot\nu)-2\left(a\cdot\nabla_x w+b\left(\partial_t w\right)+cw\right)\left(\left(\nabla_x w\right)\cdot\nu\right).$$

Then by (A.1), (A.2) and (A.8), we have

$$|Q\cdot\nu|\leq C_3\left(\left|\nabla_{x,t}w\right|^2+w^2\right),\quad (x,t)\in S_T.\qquad\text{(A.16)}$$

Here and henceforth, $C_k(k=3,4,\dots)$ denote generic positive constants which may depend on x_1^0, r, T, n, β, ρ, s_0, and s_1, but are independent of s. It follows from (A.11), (A.12), (A.14), (A.15) and (A.16) that

$$\begin{aligned}
&2\int_{G_T}(\Box w)\left(cw+b\left(\partial_t w\right)+a\cdot\nabla_x w\right)\mathrm{d}x\mathrm{d}t\\
&\geq\frac{1}{2}r\int_{\Sigma_0\bigcup\Sigma_T}\left(\left(\partial_t w+\partial_1 w\right)^2+\left|\nabla_{x'}w\right|^2\right)\mathrm{d}x\\
&\quad-c\int_{\partial\Sigma_0\bigcup\partial\Sigma_T}w^2\mathrm{d}\sigma-C_3\int_{S_T}\left(\left|\nabla_{x,t}w\right|^2+w^2\right)\mathrm{d}\sigma\mathrm{d}t\\
&\quad+\frac{1}{2}\int_{G_T}\left(\left(\rho-2\beta\right)\left(\partial_t w\right)^2+(4-\rho)\left|\nabla_x w\right|^2\right)\mathrm{d}x\mathrm{d}t,
\end{aligned}$$

where $\partial\Sigma_0$ and $\partial\Sigma_T$ denote the boundaries of Σ_0 and Σ_T, respectively, and $\mathrm{d}\sigma$ is an area element of $\partial\Omega$. Furthermore, as (4.1.40) in [17], we can show that

$$\int_{\partial\Sigma_0 \bigcup \partial\Sigma_T} w^2 \mathrm{d}\sigma \le T \int_{S_T} \left(w_t^2 + \frac{3}{T^2} w^2 \right) \mathrm{d}\sigma \mathrm{d}t.$$

Therefore,

$$\begin{aligned}
&2\int_{G_T} (\Box w)\left(cw + b\left(\partial_t w\right) + a\cdot\nabla_x w\right)\mathrm{d}x\mathrm{d}t \\
&\ge \frac{1}{2} r \int_{\Sigma_0 \bigcup \Sigma_T} \left(\left(\partial_t w + \partial_1 w\right)^2 + \left|\nabla_{x'} w\right|^2 \right) \mathrm{d}x \\
&\quad - C_4 \int_{S_T} \left(\left|\nabla_{x,t} w\right|^2 + w^2 \right) \mathrm{d}\sigma\mathrm{d}t \\
&\quad + \frac{1}{2} \int_{G_T} \left(\left(\rho - 2\beta\right)\left(\partial_t w\right)^2 + \left(4-\rho\right)\left|\nabla_x w\right|^2 \right) \mathrm{d}x\mathrm{d}t.
\end{aligned} \tag{A.17}$$

Moreover, we can verify that

$$\begin{aligned}
&2dw\left(cw + b\left(\partial_t w\right) + a\cdot\nabla_x w\right) \\
&= \nabla_x \cdot \left(dw^2 a\right) + \partial_t\left(dbw^2\right) - w^2\left(\nabla_x\cdot(da)\right) - w^2\partial_t\left(bd\right) + 2dcw^2.
\end{aligned}$$

Then we have

$$\begin{aligned}
&2\int_{G_T} dw\left(cw + b\left(\partial_t w\right) + a\cdot\nabla_x w\right)\mathrm{d}x\mathrm{d}t \\
=& \int_{\Sigma_T} dw^2\left(b - a_1\right)\mathrm{d}x + \int_{\Sigma_0} dw^2\left(a_1 - b\right)\mathrm{d}x \\
&+ \int_{S_T} dw^2\left(a\cdot\nu\right)\mathrm{d}\sigma\mathrm{d}t + \int_{G_T} w^2\left(2dc - \nabla_x\cdot(da) - \partial_t(bd)\right)\mathrm{d}x\mathrm{d}t.
\end{aligned} \tag{A.18}$$

By (1.5), (A.1) and (A.6), we have

$$\begin{aligned}
d &= \frac{1}{16}\beta^2\left(x_1 - x_1^0 + \frac{T}{2}\right)^2 - \frac{1}{4}|x|^2 \ge \frac{1}{16}\beta^2\left(\frac{T}{2} - r\right)^2 - \frac{1}{4}\left(r + x_1^0\right)^2 \\
&> \frac{1}{4}\left(2r + x_1^0\right)^2 - \frac{1}{4}\left(r + x_1^0\right)^2 = \frac{1}{4}\left(3r^2 + 2x_1^0 r\right) \ge \frac{5}{4}r^2, \quad (x,t)\in\Sigma_T.
\end{aligned}$$

By (A.1) and (A.6), we have

$$b - a_1 = \frac{1}{2}\beta\left(x_1 - x_1^0 + \frac{T}{2}\right) - x_1 \ge \frac{1}{2}\beta\left(\frac{T}{2} - r\right) - \left(x_1^0 + r\right) > r, \quad (x,t)\in\Sigma_T. \tag{A.19}$$

Therefore,

$$dw^2\left(b - a_1\right) \ge \frac{5}{4}r^3 w^2, \quad (x,t)\in\Sigma_T. \tag{A.20}$$

Similarly, we have

$$\begin{aligned}
d &= \frac{1}{16}\beta^2\left(x_1 - x_1^0 - \frac{T}{2}\right)^2 - \frac{1}{4}|x|^2 \\
&\geq \frac{1}{16}\beta^2\left(\frac{T}{2} - r\right)^2 - \frac{1}{4}\left(r + x_1^0\right)^2 \geq \frac{5}{4}r^2, \quad (x,t) \in \Sigma_0,
\end{aligned}$$

and

$$\begin{aligned}
a_1 - b &= x_1 - \frac{1}{2}\beta\left(x_1 - x_1^0 - \frac{T}{2}\right) \geq \left(x_1^0 - r\right) - \frac{1}{2}\beta\left(r - \frac{T}{2}\right) \\
&> \left(x_1^0 - r\right) + \left(2r + x_1^0\right) = 2x_1^0 + r \geq 3r, \quad (x,t) \in \Sigma_0.
\end{aligned} \tag{A.21}$$

Therefore,

$$dw^2\left(a_1 - b\right) \geq \frac{15}{4}r^3 w^2, \quad (x,t) \in \Sigma_0. \tag{A.22}$$

Furthermore we can verify that

$$\begin{aligned}
&2dc - \nabla_x \cdot (da) - \partial_t (bd) \\
&= \frac{1}{2}\left(|x|^2 - \frac{1}{16}\rho\beta^2\left(t - x_1^0 - \frac{T}{2}\right)^2 + \frac{1}{4}\rho|x|^2 - \frac{1}{8}\beta^3\left(t - x_1^0 - \frac{T}{2}\right)^2\right).
\end{aligned}$$

Then by (A.1), (A.2) and (A.5), we have

$$\begin{aligned}
&2dc - \nabla_x \cdot (da) - \partial_t (bd) \\
&\geq \frac{1}{2}\left\{\left(x_1^0 - r\right)^2 - \frac{1}{16}\beta^2\left(\frac{T}{2} + r\right)^2\left(\frac{64\left(x_1^0 - r\right)^2}{\beta^2(T+2r)^2} - 2\beta\right)\right. \\
&\left.+\frac{1}{4}\rho\left(x_1^0 - r\right)^2 - \frac{1}{8}\beta^3\left(\frac{T}{2} + r\right)^2\right\} \\
&= \frac{1}{8}\rho\left(x_1^0 - r\right)^2, \quad (x,t) \in G_T.
\end{aligned} \tag{A.23}$$

It follows from (A.18), (A.20), (A.22) and (A.23) that

$$\begin{aligned}
&2\int_{G_T} dw\left(cw + b\left(\partial_t w\right) + a \cdot \nabla_x w\right) \mathrm{d}x\mathrm{d}t \\
&\geq \frac{5}{4}r^3 \int_{\Sigma_0 \bigcup \Sigma_T} w^2 \mathrm{d}x + \frac{1}{8}\rho\left(x_1^0 - r\right)^2 \int_{G_T} w^2 \mathrm{d}x\mathrm{d}t - C_5 \int_{S_T} w^2 \mathrm{d}\sigma\mathrm{d}t.
\end{aligned} \tag{A.24}$$

Furthermore, by (A.8), (A.18), (A.19) and (A.21), we have

$$\begin{aligned}
&2\int_{G_T} w\left(cw + b\left(\partial_t w\right) + a \cdot \nabla_x w\right) \mathrm{d}x\mathrm{d}t \\
&= \int_{\Sigma_T} \left(b - a_1\right) w^2 \mathrm{d}x + \int_{\Sigma_0} \left(a_1 - b\right) w^2 \mathrm{d}x \\
&\quad + \int_{S_T} w^2 \left(a \cdot \nu\right) \mathrm{d}\sigma\mathrm{d}t + \int_{G_T} \left(2c - \nabla_x \cdot a - \partial_t b\right) w^2 \mathrm{d}x\mathrm{d}t \\
&\geq r\int_{\Sigma_0 \bigcup \Sigma_T} w^2 \mathrm{d}x - \frac{1}{2}\rho \int_{G_T} w^2 \mathrm{d}x\mathrm{d}t - C_6 \int_{S_T} w^2 \mathrm{d}\sigma\mathrm{d}t.
\end{aligned} \tag{A.25}$$

Hence, by (A.7), (A.17), (A.24) and (A.25), there exists $s_1 > 0$ such that, for all $s \geq s_1$,

$$\begin{aligned}
&\int_{G_T} (\Box v)^2 \mathrm{e}^{2s\varphi} \mathrm{d}x\mathrm{d}t = \int_{G_T} (Lw)^2 \mathrm{d}x\mathrm{d}t \\
&\geq \min\left(\frac{r}{2}, \frac{5}{4}r^3\right) \int_{\Sigma_0 \bigcup \Sigma_T} \Big(s\left(\partial_t w + \partial_1 w\right)^2 + s\left|\nabla_{x'} w\right|^2 + s^3 w^2 \Big) \mathrm{d}x \\
&\quad + \frac{1}{2} \int_{G_T} \Big((\rho - 2\beta) s \left(\partial_t w\right)^2 + (4-\rho) s \left|\nabla_x w\right|^2 \\
&\qquad + \frac{1}{8} \rho \left(x_1^0 - r\right)^2 s^3 w^2 \Big) \mathrm{d}x\mathrm{d}t - C_7 \int_{S_T} \Big(s\left|\nabla_{x,t} w\right|^2 + s^3 w^2 \Big) \mathrm{d}\sigma\mathrm{d}t.
\end{aligned} \tag{A.26}$$

Furthermore, by (1.5) and (A.5), we see that

$$\min\left(\frac{r}{2}, \frac{5}{4}r^3\right) > 0, \quad \rho - 2\beta > 0, \quad 4 - \rho > 0, \quad \text{and} \quad \rho\left(x_1^0 - r\right)^2 > 0. \tag{A.27}$$

On the other hand, by $w = \mathrm{e}^{s\varphi} v$, we have

$$\nabla_{x,t} w = s\mathrm{e}^{s\varphi} v \left(\nabla_{x,t}\varphi\right) + \mathrm{e}^{s\varphi}\left(\nabla_{x,t} v\right) = s\left(\nabla_{x,t}\varphi\right) w + \mathrm{e}^{s\varphi}\left(\nabla_{x,t} v\right).$$

Therefore, we have

$$\left(\partial_t v + \partial_1 v\right)^2 \mathrm{e}^{2s\varphi} \leq 2s^2 \left(\partial_t \varphi + \partial_1 \varphi\right)^2 w^2 + 2\left(\partial_t w + \partial_1 w\right)^2,$$

$$\left(\partial_t v\right)^2 \mathrm{e}^{2s\varphi} \leq 2s^2 \left(\partial_t \varphi\right)^2 w^2 + 2\left(\partial_t w\right)^2,$$

$$\left|\nabla_x v\right|^2 \mathrm{e}^{2s\varphi} \leq 2s^2 \left|\nabla_x \varphi\right|^2 w^2 + 2\left|\nabla_x w\right|^2,$$

$$\left|\nabla_{x'} v\right|^2 \mathrm{e}^{2s\varphi} \leq 2s^2 \left|\nabla_{x'} \varphi\right|^2 w^2 + 2\left|\nabla_{x'} w\right|^2,$$

and

$$\left|\nabla_{x,t} w\right|^2 \leq 2\left|\nabla_{x,t} v\right|^2 \mathrm{e}^{2s\varphi} + 2s^2 \left|\nabla_{x,t}\varphi\right|^2 \mathrm{e}^{2s\varphi} v^2.$$

Using (A.26), (A.27), and the above inequalities, we have

$$
\begin{aligned}
&\int_{G_T} \left(s\left|\nabla_{x,t}v\right|^2 + s^3v^2\right) \mathrm{e}^{2s\varphi}\mathrm{d}x\mathrm{d}t \\
&\quad + \int_{\Sigma_0 \bigcup \Sigma_T} \left(s\left(\partial_t v + \partial_1 v\right)^2 + s\left|\nabla_{x'}v\right|^2 + s^3v^2\right) \mathrm{e}^{2s\varphi}\mathrm{d}x \\
&\le C_8 \left\{ \int_{G_T} \left(s\left|\nabla_{x,t}w\right|^2 + s^3w^2\right) \mathrm{d}x\mathrm{d}t \right. \\
&\quad \left. + \int_{\Sigma_0 \bigcup \Sigma_T} \left(s\left(\partial_t w + \partial_1 w\right)^2 + s\left|\nabla_{x'}w\right|^2 + s^3w^2\right) \mathrm{d}x \right\} \\
&\le C_9 \left\{ \int_{S_T} \left(s\left|\nabla_{x,t}w\right|^2 + s^3w^2\right) \mathrm{d}\sigma\mathrm{d}t + \int_{G_T} \left(\Box v\right)^2 \mathrm{e}^{2s\varphi}\mathrm{d}x\mathrm{d}t \right\} \\
&\le C_{10} \left\{ \int_{S_T} \left(s\left|\nabla_{x,t}v\right|^2 + s^3v^2\right) \mathrm{e}^{2s\varphi}\mathrm{d}\sigma\mathrm{d}t + \int_{G_T} \left(\Box v\right)^2 \mathrm{e}^{2s\varphi}\mathrm{d}x\mathrm{d}t \right\}
\end{aligned}
$$

We have completed the proof of Lemma 2.1. □

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