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Abstract. We write $K_1 \geq K_2$ for two prime knots K_1, K_2 if there exists a surjective group homomorphism from $G(K_1)$ onto $G(K_2)$ where $G(K_1), G(K_2)$ are the knot groups of K_1, K_2 respectively. In this paper, we determine this partial order in Rolfsen's knot table.

1. Introduction

Let K be a prime knot and $G(K)$ its knot group. It is well known that a partial order can be defined on the set of prime knots as follows. For two knots K_1, K_2, we write $K_1 \geq K_2$ if there exists a surjective group homomorphism from $G(K_1)$ onto $G(K_2)$.

In this paper, we determine this partial order "$\geq$" in Rolfsen's knot table, which lists all the prime knots of 10 crossings or less. That is to say, the following is the main result of this paper. The numbering of the knots follows that of Rolfsen's book [8].

Theorem 1.1. *The above partial order in Rolfsen's knot table is given as below:*

$$\begin{matrix} 8_5, 8_{10}, 8_{15}, 8_{18}, 8_{19}, 8_{20}, 8_{21}, 9_1, 9_6, 9_{16}, 9_{23}, 9_{24}, 9_{28}, 9_{40}, \\ 10_5, 10_9, 10_{32}, 10_{40}, 10_{61}, 10_{62}, 10_{63}, 10_{64}, 10_{65}, 10_{66}, 10_{76}, 10_{77}, 10_{78}, 10_{82}, \\ 10_{84}, 10_{85}, 10_{87}, 10_{98}, 10_{99}, 10_{103}, 10_{106}, 10_{112}, 10_{114}, 10_{139}, 10_{140}, 10_{141}, \\ 10_{142}, 10_{143}, 10_{144}, 10_{159}, 10_{164} \end{matrix} \quad \geq 3_1,$$

$$8_{18}, 9_{37}, 9_{40}, 10_{58}, 10_{59}, 10_{60}, 10_{122}, 10_{136}, 10_{137}, 10_{138} \quad \geq 4_1,$$

$$10_{74}, 10_{120}, 10_{122} \geq 5_2.$$

Here we mention the contexts of this paper. In Section 2, we construct explicitly a surjective homomorphism for any pair of knots which belongs to the above list. Therefore we can prove that there exists such a partial order. In Section 3, we give the definition and results of the twisted Alexander invariants for knots. In Section 4, we state non-existence of surjective homomorphisms by using the twisted Alexander invariants. It finishes the proof of Theorem 1.1. In Section 5, we give the tables of data which are needed to prove the main theorem.

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2. Construction of surjective homomorphisms

In this section, we construct a surjective homomorphism between two knots which appears in the list of the main theorem. For any knot K, we always take and denote a Wiritinger presentation of its knot group $G(K)$ as follows:

$$G(K) = \langle x_1, \dots, x_n \,|\, r_1, \dots, r_{n-1} \rangle.$$

We denote by $\bar{x}$ the inverse of x in $G(K)$. Further we write a number for a generator of $G(K)$ in Table 1, 2 and 3 for simplicity. For example, we write $1, 2, \dots, 9, 10$ for the generators $x_1, x_2, \dots, x_9, x_{10}$ and $12\bar{1}\overline{10}$ means a relator $x_1 x_2 \bar{x}_1 \bar{x}_{10}$.

Proposition 2.1. *There exist a surjective homomorphism $G(K_1) \to G(K_2)$ for any pair (K_1, K_2) of knots in Theorem 1.1.*

Proof. First, we consider surjective homomorphism onto the knot group of the trefoil knot 3_1. The knot group of 3_1 admits a presentation:

$$G(3_1) = \langle x_1, x_2, x_3 \,|\, x_3 x_1 \bar{x}_3 \bar{x}_2,\ x_1 x_2 \bar{x}_1 \bar{x}_3 \rangle.$$

Table 1 gives the relators of each knot group $G(K)$ and the images of generators of $G(K)$ onto $G(3_1)$. We can check easily that the mappings are surjective homomorphisms.

Next, we construct surjective homomorphisms onto the knot group of the figure eight knot 4_1. The knot group of 4_1 has a presentation:

$$G(4_1) = \langle x_1, x_2, x_3, x_4 \,|\, x_4 x_2 \bar{x}_4 \bar{x}_1,\ x_1 x_2 \bar{x}_1 \bar{x}_3,\ x_2 x_4 \bar{x}_2 \bar{x}_3 \rangle.$$

Similarly, Table 2 gives surjective homomorphisms to $G(4_1)$.

Finally, we fix a presentation of $G(5_2)$:

$$G(5_2) = \langle x_1, x_2, x_3, x_4, x_5 \,|\, x_4 x_1 \bar{x}_4 \bar{x}_2,\ x_5 x_2 \bar{x}_5 \bar{x}_3,\ x_2 x_3 \bar{x}_2 \bar{x}_4,\ x_1 x_4 \bar{x}_1 \bar{x}_5 \rangle.$$

and surjective homomorphisms to $G(5_2)$ are described in Table 3. □

3. Twisted Alexander invariants of knots

In this section, we recall briefly the definition and some properties of the twisted Alexander invariants for knots. See [9] and [5] for more precise definition in general cases of finitely presentable groups.

Let us take a Wirtinger presentation of a knot group $G(K)$ as follows:

$$G(K) = \langle x_1, x_2, \dots, x_u \,|\, r_1, r_2, \dots, r_{u-1} \rangle.$$

In this paper, the integers $\mathbb{Z}$ can be identified with the cyclic group $\langle t \rangle$ as a multiplicative group. Then by mapping each generator x_i to t, the abelianization

$$\alpha : G(K) \to \mathbb{Z} \simeq \langle t \rangle$$

is obtained. Now we fix a prime integer p and take a representation

$$\rho : G(K) \to SL(2; \mathbb{F}_p).$$

Here $\mathbb{F}_p$ is the finite field $\mathbb{Z}/p\mathbb{Z}$. Two maps ρ and α induce ring homomorphisms $\tilde{\rho} : \mathbb{Z}[G(K)] \to M(2; \mathbb{F}_p)$ and $\tilde{\alpha} : \mathbb{Z}[G(K)] \to \mathbb{Z}[t, t^{-1}]$ respectively. Then we get the tensor representation

$$\tilde{\rho} \otimes \tilde{\alpha} : \mathbb{Z}[G(K)] \to M(2; \mathbb{F}_p[t, t^{-1}]).$$

From the fixed Wiritinger presentation, a natural homomorphism $\mathbb{Z}[F_u] \to \mathbb{Z}[G(K)]$ is induced where F_u is the free group on generators $\{x_1, \dots, x_u\}$. Then a ring homomorphism

$$\Phi : \mathbb{Z}[F_u] \to M(2; \mathbb{F}_p[t, t^{-1}])$$

is defined by the composite of the above natural homomorhpism and $\tilde{\rho} \otimes \tilde{\alpha}$.

Now we define the matrix

$$M \in M((u-1) \times u; M(2; \mathbb{F}_p[t, t^{-1}]))$$

to be the $(u-1) \times u$ matrix whose (i, j)-component is $\Phi\left(\frac{\partial r_i}{\partial x_j}\right) \in M(2; \mathbb{F}_p[t, t^{-1}])$. Here $\partial/\partial x_j$ denotes the Fox derivation $\partial/\partial x_j : \mathbb{Z}[F_u] \to \mathbb{Z}[F_u]$ for each x_j.

Further we write M_1 for the $(u-1) \times (u-1)$ matrix obtained from M removing the first column. This matrix M_1 can be considered as a $2(u-1) \times 2(u-1)$-matrix whose entries belong to $\mathbb{F}_p[t, t^{-1}]$. Here let $\Delta^{(n)}_{K,\rho}(t)$ denotes the determinant of M_1 and $\Delta^{(d)}_{K,\rho}(t)$ the determinant of $\Phi(x_1 - 1)$. By using these polynomials, now we define the following.

Definition 3.1. The twisted Alexander invariant of $G(K)$ for a representation $\rho : G(K) \to SL(2; \mathbb{F}_p)$ is defined to be

$$\Delta_{K,\rho}(t) = \frac{\Delta^{(n)}_{K,\rho}(t)}{\Delta^{(d)}_{K,\rho}(t)} = \frac{\det M_1}{\det \Phi(x_1 - 1)}.$$

Remark 3.2. It does not depend on the choices of Wiritinger presentations, up to a factor t^k $(k \in \mathbb{Z})$. Namely, the twisted Alexander invariant $\Delta_{K,\rho}(t)$ is well-defined as an invariant of a triple (K, ρ, α). See [9] as a reference.

By using the twisted Alexander invariants, a criterion for existence of a surjective homomorphism between two knot groups is given as follows. It is proved for more general cases in [5]. Let K_1 and K_2 be two knots and α_1, α_2 surjective homomorphisms from the knot groups $G(K_1), G(K_2)$ to $\mathbb{Z}$ respectively. Suppose that there exists a surjective homomorphism $\varphi : G(K_1) \to G(K_2)$ such that $\alpha_1 = \alpha_2 \circ \varphi$.

Theorem 3.3 (Kitano-Suzuki-Wada)**.** *For any representation $\rho_2 : G(K_2) \to SL(2; \mathbb{F}_p)$ and $\rho_1 = \rho_2 \circ \varphi$, $\Delta_{K_1,\rho_1}(t)$ is divisible by $\Delta_{K_2,\rho_2}(t)$. More precisely, $\Delta^{(n)}_{K_1,\rho_1}(t)$ is divisible by $\Delta^{(n)}_{K_2,\rho_2}(t)$ and $\Delta^{(d)}_{K_1,\rho_1}(t) = \Delta^{(d)}_{K_2,\rho_2}(t)$.*

Remark 3.4. The corresponding fact on the classical Alexander polynomial is well known. Namely, if there exists a surjective homomorphism from $G(K_1)$ to $G(K_2)$, then the Alexander polynomial of K_1 is divisible by that of K_2. See [1] as a reference.

4. Non-existence of surjective homomorphisms

In this section, we prove non-existence of surjective homomorphisms between two knots except for the pairs in Theorem 1.1.

First we consider the pairs of knots which does not appear in the main theorem and in Table 4. By using only the classical Alexander polynomial, we can show easily that there exists no surjective homomorphism between them. Therefore we check whether there exist a surjective homomorphism or not, only for any pair of knots in Table 4.

The following theorem is obtained from Theorem 3.3 as a direct consequence.

Theorem 4.1. *If there exists a representation $\rho_2 : G(K_2) \to SL(2;\mathbb{F}_p)$ such that for any representation $\rho_1 : G(K_1) \to SL(2;\mathbb{F}_p)$, $\Delta^{(n)}_{K_1,\rho_1}(t)$ is not divided by $\Delta^{(n)}_{K_2,\rho_2}(t)$ or $\Delta^{(d)}_{K_2,\rho_2}(t) \neq \Delta^{(d)}_{K_1,\rho_1}(t)$, then there exists no surjective homomorphism from $G(K_1)$ onto $G(K_2)$.*

By applying this theorem with aid of computer, we can prove that there exist no surjective homomorphism between any pair of knots in Table 4. This completes the proof of Theorem 1.1.

Here we mention how to read Table 4 and Table 5. First there are some numbers of knots with a prime integer in each row of Table 4. For example, $8_{11}(5)$ in the row of 3_1 means that non-existence of a surjective homomorphism from $G(8_{11})$ onto $G(3_1)$ is checked by using the twisted Alexander invariants of $SL(2;\mathbb{F}_5)$-representation.

All twisted Alexander invariants which we use to check Table 4 are listed in Table 5. Each row of Table 5 gives a knot K, a prime number p and all pairs of the numerator $\Delta^{(n)}_{K,\rho}(t)$ and denominator $\Delta^{(d)}_{K,\rho}(t)$ of the twisted Alexander invariants. We note that the twisted Alexander invariants are invariant under changing a representation to any conjugate representation in the $SL(2;\mathbb{F}_p)$-representations. Therefore we consider only the classes of conjugate representations.

By the similar argument of [4] and [3], it is easily proved that the twisted Alexander invariant for an $SL(2;\mathbb{F}_p)$-representation of a knot is symmetric up to a factor t^k. It is clear that its denominator is symmetric, because it is the characteristic polynomial of the matrix $\rho(x_1)$. Hence its numerator is also symmetric. Totally we obtain

$$\Delta^{(d)}_{K,\rho}(t) = t^{k_1}\Delta^{(d)}_{K,\rho}(t^{-1}),\ \Delta^{(n)}_{K,\rho}(t) = t^{k_2}\Delta^{(n)}_{K,\rho}(t^{-1}).$$

Therefore in Table 5, a symbol $a_0 + a_1 + a_2 + \cdots + a_n$ represents a symmetric polynomial $a_0 + a_1(t^{-1}+t) + a_2(t^{-2}+t^2) + \cdots + a_n(t^{-n}+t^n)$.

Remark 4.2. All values of twisted Alexander invariants in Table 5 are calculated by second author's computer program and a part of them (numerators of the twisted Alexander invariants) are also done by Kodama Knot program [6].

5. Tables

Table 1: Surjective homomorphism to 3_1

K	relators
	surjective homomorphism to 3_1
8_5	$72\overline{71}, 83\overline{82}, 64\overline{63}, 15\overline{14}, 36\overline{35}, 47\overline{46}, 28\overline{27}$
	$1 \mapsto 3, 2 \mapsto 2, 3 \mapsto 1, 4 \mapsto 3, 5 \mapsto 3, 6 \mapsto 2, 7 \mapsto 1, 8 \mapsto 3$
8_{10}	$72\overline{71}, 42\overline{43}, 63\overline{64}, 85\overline{84}, 35\overline{36}, 17\overline{16}, 28\overline{27}$
	$1 \mapsto 3, 2 \mapsto 1, 3 \mapsto 2, 4 \mapsto 3, 5 \mapsto 3, 6 \mapsto 1, 7 \mapsto 13\overline{1}, 8 \mapsto 3$
8_{15}	$41\overline{42}, 82\overline{83}, 53\overline{54}, 24\overline{25}, 75\overline{76}, 16\overline{17}, 37\overline{38}$
	$1 \mapsto 1, 2 \mapsto 3, 3 \mapsto 3, 4 \mapsto 13\overline{1}, 5 \mapsto 1, 6 \mapsto 2, 7 \mapsto 3, 8 \mapsto 3$
8_{18}	$41\overline{42}, 53\overline{52}, 63\overline{64}, 75\overline{74}, 85\overline{86}, 17\overline{16}, 27\overline{28}$
	$1 \mapsto 1, 2 \mapsto 2, 3 \mapsto 1, 4 \mapsto 3, 5 \mapsto 3, 6 \mapsto 13\overline{1}, 7 \mapsto 3, 8 \mapsto 1$

Continued on next page

Table 1 Continued from previous page

K	relators / surjective homomorphism to 3_1
8_{19}	$52\bar{5}\bar{1}, 83\bar{8}\bar{2}, 64\bar{6}\bar{3}, 15\bar{1}\bar{4}, 36\bar{3}\bar{5}, 17\bar{1}\bar{6}, 58\bar{5}\bar{7}$
	$1 \mapsto 3, 2 \mapsto 3, 3 \mapsto 1, 4 \mapsto 3, 5 \mapsto 3, 6 \mapsto 2, 7 \mapsto 1, 8 \mapsto 13\bar{1}$
8_{20}	$51\bar{5}\bar{2}, 72\bar{7}\bar{3}, 13\bar{1}\bar{4}, 75\bar{7}\bar{4}, 35\bar{3}\bar{6}, 47\bar{4}\bar{6}, 58\bar{5}\bar{7}$
	$1 \mapsto 2, 2 \mapsto \bar{2}32, 3 \mapsto 3, 4 \mapsto 1, 5 \mapsto 3, 6 \mapsto 3, 7 \mapsto 2, 8 \mapsto 1$
8_{21}	$81\bar{8}\bar{2}, 73\bar{7}\bar{2}, 13\bar{1}\bar{4}, 74\bar{7}\bar{5}, 16\bar{1}\bar{5}, 86\bar{8}\bar{7}, 57\bar{5}\bar{8}$
	$1 \mapsto 2, 2 \mapsto 3, 3 \mapsto 3, 4 \mapsto 1, 5 \mapsto 2, 6 \mapsto 2, 7 \mapsto 3, 8 \mapsto 1$
9_1	$61\bar{6}\bar{2}, 72\bar{7}\bar{3}, 83\bar{8}\bar{4}, 94\bar{9}\bar{5}, 15\bar{1}\bar{6}, 26\bar{2}\bar{7}, 37\bar{3}\bar{8}, 48\bar{4}\bar{9}$
	$1 \mapsto 1, 2 \mapsto 2, 3 \mapsto 3, 4 \mapsto 1, 5 \mapsto 2, 6 \mapsto 3, 7 \mapsto 1, 8 \mapsto 2, 9 \mapsto 3$
9_6	$71\bar{7}\bar{2}, 82\bar{8}\bar{3}, 93\bar{9}\bar{4}, 64\bar{6}\bar{5}, 15\bar{1}\bar{6}, 46\bar{4}\bar{7}, 27\bar{2}\bar{8}, 38\bar{3}\bar{9}$
	$1 \mapsto 1, 2 \mapsto 2, 3 \mapsto 3, 4 \mapsto 1, 5 \mapsto \bar{1}21, 6 \mapsto 2, 7 \mapsto 3, 8 \mapsto 1, 9 \mapsto 2$
9_{16}	$82\bar{8}\bar{1}, 93\bar{9}\bar{2}, 64\bar{6}\bar{3}, 75\bar{7}\bar{4}, 16\bar{1}\bar{5}, 47\bar{4}\bar{6}, 38\bar{3}\bar{7}, 29\bar{2}\bar{8}$
	$1 \mapsto 1, 2 \mapsto 3, 3 \mapsto 2, 4 \mapsto \bar{1}21, 5 \mapsto 1, 6 \mapsto 1, 7 \mapsto 2, 8 \mapsto 2, 9 \mapsto 1$
9_{23}	$31\bar{3}\bar{2}, 62\bar{6}\bar{3}, 13\bar{1}\bar{4}, 84\bar{8}\bar{5}, 25\bar{2}\bar{6}, 46\bar{4}\bar{7}, 97\bar{9}\bar{8}, 58\bar{5}\bar{9}$
	$1 \mapsto 1, 2 \mapsto 21\bar{2}, 3 \mapsto 2, 4 \mapsto 3, 5 \mapsto 2, 6 \mapsto 1, 7 \mapsto 2, 8 \mapsto \bar{2}32, 9 \mapsto 3$
9_{24}	$42\bar{4}\bar{1}, 53\bar{5}\bar{2}, 73\bar{7}\bar{4}, 25\bar{2}\bar{4}, 95\bar{9}\bar{6}, 87\bar{8}\bar{6}, 37\bar{3}\bar{8}, 18\bar{1}\bar{9}$
	$1 \mapsto 1, 2 \mapsto 3, 3 \mapsto 2, 4 \mapsto 2, 5 \mapsto 1, 6 \mapsto 2, 7 \mapsto 2, 8 \mapsto 2, 9 \mapsto 3$
9_{28}	$62\bar{6}\bar{1}, 52\bar{5}\bar{3}, 23\bar{2}\bar{4}, 95\bar{9}\bar{4}, 35\bar{3}\bar{6}, 86\bar{8}\bar{7}, 17\bar{1}\bar{8}, 49\bar{4}\bar{8}$
	$1 \mapsto 1, 2 \mapsto 1, 3 \mapsto 2, 4 \mapsto 3, 5 \mapsto 3, 6 \mapsto 1, 7 \mapsto 2, 8 \mapsto 3, 9 \mapsto 3$
9_{40}	$81\bar{8}\bar{2}, 73\bar{7}\bar{2}, 63\bar{6}\bar{4}, 24\bar{2}\bar{5}, 16\bar{1}\bar{5}, 96\bar{9}\bar{7}, 57\bar{5}\bar{8}, 49\bar{4}\bar{8}$
	$1 \mapsto 1, 2 \mapsto 2, 3 \mapsto 2, 4 \mapsto 3, 5 \mapsto 1, 6 \mapsto 1, 7 \mapsto 2, 8 \mapsto 3, 9 \mapsto 3$
10_5	$71\bar{7}\bar{2}, 83\bar{8}\bar{2}, 94\bar{9}\bar{3}, 105\bar{10}\bar{4}, 26\bar{2}\bar{5}, 16\bar{1}\bar{7}, 67\bar{6}\bar{8}, 39\bar{3}\bar{8}, 410\bar{4}\bar{9}$
	$1 \mapsto 2, 2 \mapsto 1, 3 \mapsto 3, 4 \mapsto 2, 5 \mapsto 1, 6 \mapsto 1, 7 \mapsto \bar{1}21, 8 \mapsto 2, 9 \mapsto 1, 10 \mapsto 3$
10_9	$92\bar{9}\bar{1}, 73\bar{7}\bar{2}, 84\bar{8}\bar{3}, 14\bar{1}\bar{5}, 105\bar{10}\bar{6}, 27\bar{2}\bar{6}, 38\bar{3}\bar{7}, 49\bar{4}\bar{8}, 59\bar{5}\bar{10}$
	$1 \mapsto 2, 2 \mapsto 1, 3 \mapsto 3, 4 \mapsto 2, 5 \mapsto 2, 6 \mapsto 3, 7 \mapsto 2, 8 \mapsto 1, 9 \mapsto 3, 10 \mapsto 1$
10_{32}	$41\bar{4}\bar{2}, 93\bar{9}\bar{2}, 84\bar{8}\bar{3}, 74\bar{7}\bar{5}, 105\bar{10}\bar{6}, 56\bar{5}\bar{7}, 17\bar{1}\bar{8}, 29\bar{2}\bar{8}, 310\bar{3}\bar{9}$
	$1 \mapsto 1, 2 \mapsto 2, 3 \mapsto 3, 4 \mapsto 3, 5 \mapsto 1, 6 \mapsto \bar{1}21, 7 \mapsto 2, 8 \mapsto 3, 9 \mapsto \bar{2}32, 10 \mapsto 2$
10_{40}	$62\bar{6}\bar{1}, 83\bar{8}\bar{2}, 93\bar{9}\bar{4}, 75\bar{7}\bar{4}, 16\bar{1}\bar{8}, 57\bar{5}\bar{6}, 28\bar{2}\bar{7}, 108\bar{10}\bar{9}, 49\bar{4}\bar{10}$
	$1 \mapsto 2, 2 \mapsto 3, 3 \mapsto 2, 4 \mapsto 1, 5 \mapsto 3, 6 \mapsto \bar{2}32, 7 \mapsto 2, 8 \mapsto 1, 9 \mapsto \bar{1}21, 10 \mapsto 2$
10_{61}	$82\bar{8}\bar{1}, 64\bar{6}\bar{2}, 74\bar{7}\bar{3}, 104\bar{10}\bar{5}, 95\bar{9}\bar{6}, 37\bar{3}\bar{6}, 18\bar{1}\bar{7}, 29\bar{2}\bar{8}, 59\bar{5}\bar{10}$
	$1 \mapsto 1, 2 \mapsto 3, 3 \mapsto 2, 4 \mapsto 1, 5 \mapsto 1, 6 \mapsto 1, 7 \mapsto 3, 8 \mapsto 2, 9 \mapsto 1, 10 \mapsto 1$
10_{62}	$62\bar{6}\bar{1}, 92\bar{9}\bar{3}, 74\bar{7}\bar{3}, 105\bar{10}\bar{4}, 16\bar{1}\bar{5}, 37\bar{3}\bar{6}, 48\bar{4}\bar{7}, 28\bar{2}\bar{9}, 89\bar{8}\bar{10}$
	$1 \mapsto 1, 2 \mapsto 1, 3 \mapsto 2, 4 \mapsto 1, 5 \mapsto 1, 6 \mapsto 1, 7 \mapsto 3, 8 \mapsto 2, 9 \mapsto 3, 10 \mapsto 1$
10_{63}	$41\bar{4}\bar{2}, 92\bar{9}\bar{3}, 73\bar{7}\bar{4}, 14\bar{1}\bar{5}, 105\bar{10}\bar{6}, 36\bar{3}\bar{7}, 67\bar{6}\bar{8}, 28\bar{2}\bar{9}, 89\bar{8}\bar{10}$
	$1 \mapsto 1, 2 \mapsto 1, 3 \mapsto 2, 4 \mapsto 1, 5 \mapsto 1, 6 \mapsto 1, 7 \mapsto 21\bar{2}, 8 \mapsto 2, 9 \mapsto 3, 10 \mapsto 1$
10_{64}	$52\bar{5}\bar{1}, 72\bar{7}\bar{3}, 83\bar{8}\bar{4}, 15\bar{1}\bar{4}, 96\bar{9}\bar{5}, 107\bar{10}\bar{6}, 37\bar{3}\bar{8}, 28\bar{2}\bar{9}, 610\bar{6}\bar{9}$
	$1 \mapsto 1, 2 \mapsto 3, 3 \mapsto 3, 4 \mapsto 3, 5 \mapsto 2, 6 \mapsto 1, 7 \mapsto 3, 8 \mapsto 3, 9 \mapsto 3, 10 \mapsto 2$
10_{65}	$81\bar{8}\bar{2}, 12\bar{1}\bar{3}, 74\bar{7}\bar{3}, 95\bar{9}\bar{4}, 106\bar{10}\bar{5}, 37\bar{3}\bar{6}, 48\bar{4}\bar{7}, 28\bar{2}\bar{9}, 610\bar{6}\bar{9}$
	$1 \mapsto 1, 2 \mapsto 21\bar{2}, 3 \mapsto 2, 4 \mapsto 1, 5 \mapsto 1, 6 \mapsto 1, 7 \mapsto 3, 8 \mapsto 2, 9 \mapsto 1, 10 \mapsto 1$

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Table 1 Continued from previous page

K	relators
	surjective homomorphism to 3_1
10_{66}	$81\bar{8}\bar{2}, 62\bar{6}\bar{3}, 103\overline{10}\bar{4}, 94\bar{9}\bar{5}, 25\bar{2}\bar{6}, 56\bar{5}\bar{7}, 17\bar{1}\bar{8}, 78\bar{7}\bar{9}, 39\bar{3}\overline{10}$
	$1 \mapsto 1, 2 \mapsto 3, 3 \mapsto 1, 4 \mapsto 1, 5 \mapsto 1, 6 \mapsto 2, 7 \mapsto 3, 8 \mapsto 1\bar{2}3, 9 \mapsto 1, 10 \mapsto 1$
10_{76}	$62\bar{6}\bar{1}, 43\bar{4}\bar{2}, 83\bar{8}\bar{4}, 25\bar{2}\bar{4}, 16\bar{1}\bar{5}, 97\bar{9}\bar{6}, 108\overline{10}\bar{7}, 38\bar{3}\bar{9}, 710\bar{7}\bar{9}$
	$1 \mapsto 1, 2 \mapsto 3, 3 \mapsto 3, 4 \mapsto 3, 5 \mapsto 3, 6 \mapsto 2, 7 \mapsto 1, 8 \mapsto 3, 9 \mapsto 3, 10 \mapsto 2$
10_{77}	$32\bar{3}\bar{1}, 73\bar{7}\bar{2}, 14\bar{1}\bar{6}, 105\overline{10}\bar{4}, 85\bar{8}\bar{6}, 27\bar{2}\bar{6}, 97\bar{9}\bar{8}, 68\bar{6}\bar{9}, 410\bar{4}\bar{9}$
	$1 \mapsto 1, 2 \mapsto 1, 3 \mapsto 1, 4 \mapsto 1, 5 \mapsto 3, 6 \mapsto 1, 7 \mapsto 1, 8 \mapsto 2, 9 \mapsto 3, 10 \mapsto 2$
10_{78}	$72\bar{7}\bar{1}, 52\bar{5}\bar{3}, 103\overline{10}\bar{4}, 64\bar{6}\bar{5}, 35\bar{3}\bar{6}, 27\bar{2}\bar{6}, 97\bar{9}\bar{8}, 18\bar{1}\bar{9}, 49\bar{4}\overline{10}$
	$1 \mapsto 1, 2 \mapsto 1, 3 \mapsto 2, 4 \mapsto 2, 5 \mapsto 3, 6 \mapsto 1, 7 \mapsto 1, 8 \mapsto 21\bar{2}, 9 \mapsto 2, 10 \mapsto 2$
10_{82}	$61\bar{6}\bar{2}, 73\bar{7}\bar{2}, 103\overline{10}\bar{4}, 14\bar{1}\bar{5}, 86\bar{8}\bar{5}, 96\bar{9}\bar{7}, 58\bar{5}\bar{7}, 29\bar{2}\bar{8}, 39\bar{3}\overline{10}$
	$1 \mapsto 1, 2 \mapsto 21\bar{2}, 3 \mapsto 1, 4 \mapsto 2, 5 \mapsto 3, 6 \mapsto 2, 7 \mapsto 2, 8 \mapsto 1, 9 \mapsto 2, 10 \mapsto 3$
10_{84}	$92\bar{9}\bar{1}, 82\bar{8}\bar{3}, 54\bar{5}\bar{3}, 15\bar{1}\bar{4}, 36\bar{3}\bar{5}, 26\bar{2}\bar{7}, 108\overline{10}\bar{7}, 68\bar{6}\bar{9}, 710\bar{7}\bar{9}$
	$1 \mapsto 3, 2 \mapsto 2, 3 \mapsto 3, 4 \mapsto \bar{2}32, 5 \mapsto 2, 6 \mapsto 1, 7 \mapsto 21\bar{2}, 8 \mapsto 1, 9 \mapsto 1, 10 \mapsto 2$
10_{85}	$72\bar{7}\bar{1}, 62\bar{6}\bar{3}, 103\overline{10}\bar{4}, 14\bar{1}\bar{5}, 85\bar{8}\bar{6}, 97\bar{9}\bar{6}, 57\bar{5}\bar{8}, 29\bar{2}\bar{8}, 39\bar{3}\overline{10}$
	$1 \mapsto 3, 2 \mapsto 2, 3 \mapsto 3, 4 \mapsto 1, 5 \mapsto 2, 6 \mapsto 1, 7 \mapsto 1, 8 \mapsto 21\bar{2}, 9 \mapsto 1, 10 \mapsto 2$
10_{87}	$61\bar{6}\bar{2}, 73\bar{7}\bar{2}, 104\overline{10}\bar{3}, 24\bar{2}\bar{5}, 85\bar{8}\bar{6}, 47\bar{4}\bar{6}, 57\bar{5}\bar{8}, 19\bar{1}\bar{8}, 310\bar{3}\bar{9}$
	$1 \mapsto 2, 2 \mapsto 3, 3 \mapsto 2, 4 \mapsto 1, 5 \mapsto 2, 6 \mapsto 1, 7 \mapsto 1, 8 \mapsto 21\bar{2}, 9 \mapsto 1, 10 \mapsto 3$
10_{98}	$91\bar{9}\bar{2}, 62\bar{6}\bar{3}, 83\bar{8}\bar{4}, 105\overline{10}\bar{4}, 25\bar{2}\bar{6}, 46\bar{4}\bar{7}, 17\bar{1}\bar{8}, 38\bar{3}\bar{9}, 510\bar{5}\bar{9}$
	$1 \mapsto 1, 2 \mapsto 2, 3 \mapsto 3, 4 \mapsto 3, 5 \mapsto 3, 6 \mapsto 1, 7 \mapsto 2, 8 \mapsto 3, 9 \mapsto 3, 10 \mapsto 3$
10_{99}	$91\bar{9}\bar{2}, 63\bar{6}\bar{2}, 103\overline{10}\bar{4}, 85\bar{8}\bar{4}, 26\bar{2}\bar{5}, 47\bar{4}\bar{6}, 17\bar{1}\bar{8}, 59\bar{5}\bar{8}, 39\bar{3}\overline{10}$
	$1 \mapsto 1, 2 \mapsto 2, 3 \mapsto 2, 4 \mapsto 3, 5 \mapsto 2, 6 \mapsto 2, 7 \mapsto 1, 8 \mapsto 1, 9 \mapsto 3, 10 \mapsto 1$
10_{103}	$41\bar{4}\bar{2}, 92\bar{9}\bar{3}, 73\bar{7}\bar{4}, 14\bar{1}\bar{5}, 86\bar{8}\bar{5}, 36\bar{3}\bar{7}, 108\overline{10}\bar{7}, 28\bar{2}\bar{9}, 610\bar{6}\bar{9},$
	$1 \mapsto 1, 2 \mapsto \bar{1}21, 3 \mapsto 1, 4 \mapsto 2, 5 \mapsto 3, 6 \mapsto 2, 7 \mapsto 3, 8 \mapsto 1,$ $9 \mapsto \bar{1}\bar{1}211, 10 \mapsto 1\bar{2}3$
10_{106}	$72\bar{7}\bar{1}, 93\bar{9}\bar{2}, 103\overline{10}\bar{4}, 84\bar{8}\bar{5}, 36\bar{3}\bar{5}, 17\bar{1}\bar{6}, 47\bar{4}\bar{8}, 29\bar{2}\bar{8}, 59\bar{5}\overline{10}$
	$1 \mapsto 3, 2 \mapsto 2, 3 \mapsto 21\bar{2}, 4 \mapsto 2, 5 \mapsto 1, 6 \mapsto 2, 7 \mapsto 1, 8 \mapsto 21\bar{2},$ $9 \mapsto 1, 10 \mapsto 1$
10_{112}	$52\bar{5}\bar{1}, 62\bar{6}\bar{3}, 103\overline{10}\bar{4}, 75\bar{7}\bar{4}, 85\bar{8}\bar{6}, 97\bar{9}\bar{6}, 17\bar{1}\bar{8}, 29\bar{2}\bar{8}, 39\bar{3}\overline{10}$
	$1 \mapsto 1, 2 \mapsto 3, 3 \mapsto 1, 4 \mapsto 1, 5 \mapsto 2, 6 \mapsto 2, 7 \mapsto 21\bar{2}, 8 \mapsto 2, 9 \mapsto 1, 10 \mapsto 1$
10_{114}	$61\bar{6}\bar{2}, 52\bar{5}\bar{3}, 74\bar{7}\bar{3}, 84\bar{8}\bar{5}, 25\bar{2}\bar{6}, 97\bar{9}\bar{6}, 107\overline{10}\bar{8}, 19\bar{1}\bar{8}, 39\bar{3}\overline{10}$
	$1 \mapsto 1, 2 \mapsto 1, 3 \mapsto 1, 4 \mapsto 3, 5 \mapsto 1, 6 \mapsto 1, 7 \mapsto 2, 8 \mapsto 2, 9 \mapsto 21\bar{2}, 10 \mapsto 2$
10_{139}	$52\bar{5}\bar{1}, 73\bar{7}\bar{2}, 84\bar{8}\bar{3}, 15\bar{1}\bar{4}, 26\bar{2}\bar{5}, 107\overline{10}\bar{6}, 38\bar{3}\bar{7}, 59\bar{5}\bar{8}, 210\bar{2}\bar{9}$
	$1 \mapsto 3, 2 \mapsto 2, 3 \mapsto 2, 4 \mapsto 2, 5 \mapsto 1, 6 \mapsto 3, 7 \mapsto 2, 8 \mapsto 2, 9 \mapsto 21\bar{2}, 10 \mapsto 1$
10_{140}	$51\bar{5}\bar{2}, 92\bar{9}\bar{3}, 83\bar{8}\bar{4}, 14\bar{1}\bar{5}, 25\bar{2}\bar{6}, 107\overline{10}\bar{6}, 18\bar{1}\bar{7}, 38\bar{3}\bar{9}, 29\bar{2}\overline{10}$
	$1 \mapsto 1, 2 \mapsto 2, 3 \mapsto 2, 4 \mapsto 2, 5 \mapsto 3, 6 \mapsto 1, 7 \mapsto 3, 8 \mapsto 2, 9 \mapsto 2, 10 \mapsto 2$
10_{141}	$82\bar{8}\bar{1}, 62\bar{6}\bar{3}, 74\bar{7}\bar{3}, 84\bar{8}\bar{5}, 15\bar{1}\bar{6}, 106\overline{10}\bar{7}, 18\bar{1}\bar{7}, 68\bar{6}\bar{9}, 310\bar{3}\bar{9}$
	$1 \mapsto 1, 2 \mapsto 1, 3 \mapsto 2, 4 \mapsto 21\bar{2}, 5 \mapsto 2, 6 \mapsto 3, 7 \mapsto 1, 8 \mapsto 1, 9 \mapsto 2, 10 \mapsto 2$
10_{142}	$52\bar{5}\bar{1}, 63\bar{6}\bar{2}, 104\overline{10}\bar{3}, 95\bar{9}\bar{4}, 26\bar{2}\bar{5}, 107\overline{10}\bar{6}, 18\bar{1}\bar{7}, 59\bar{5}\bar{8}, 410\bar{4}\bar{9}$
	$1 \mapsto 1, 2 \mapsto 3, 3 \mapsto 2, 4 \mapsto 2, 5 \mapsto 2, 6 \mapsto 1, 7 \mapsto 3, 8 \mapsto 2, 9 \mapsto 2, 10 \mapsto 2$

Continued on next page

Table 1 Continued from previous page

K	relators
	surjective homomorphism to 3_1
10_{143}	$51\bar{5}\bar{2}, 93\bar{9}\bar{2}, 63\bar{6}\bar{4}, 84\bar{8}\bar{5}, 25\bar{2}\bar{6}, 16\bar{1}\bar{7}, 47\bar{4}\bar{8}, 19\bar{1}\bar{8}, 310\bar{3}\bar{9}$
	$1 \mapsto 1, 2 \mapsto 1, 3 \mapsto 2, 4 \mapsto 3, 5 \mapsto 1, 6 \mapsto 1, 7 \mapsto 1, 8 \mapsto 2, 9 \mapsto 21\bar{2}, 10 \mapsto 1$
10_{144}	$81\bar{8}\bar{2}, 62\bar{6}\bar{3}, 94\bar{9}\bar{3}, 85\bar{8}\bar{4}, 75\bar{7}\bar{6}, 36\bar{3}\bar{7}, 17\bar{1}\bar{8}, 49\bar{4}\bar{8}, 310\bar{3}\bar{9}$
	$1 \mapsto 2, 2 \mapsto 3, 3 \mapsto 1, 4 \mapsto 1, 5 \mapsto 1, 6 \mapsto 2, 7 \mapsto 3, 8 \mapsto 1, 9 \mapsto 1, 10 \mapsto 1$
10_{159}	$71\bar{7}\bar{2}, 92\bar{9}\bar{3}, 13\bar{1}\bar{4}, 25\bar{2}\bar{4}, 35\bar{3}\bar{6}, 47\bar{4}\bar{6}, 57\bar{5}\bar{8}, 28\bar{2}\bar{9}, 39\bar{3}\bar{10}$
	$1 \mapsto 3, 2 \mapsto 1, 3 \mapsto 1, 4 \mapsto 2, 5 \mapsto 21\bar{2}, 6 \mapsto 2, 7 \mapsto 2, 8 \mapsto 1, 9 \mapsto 1, 10 \mapsto 1$
10_{164}	$52\bar{5}\bar{1}, 73\bar{7}\bar{2}, 83\bar{8}\bar{4}, 25\bar{2}\bar{4}, 16\bar{1}\bar{5}, 97\bar{9}\bar{6}, 107\bar{10}\bar{8}, 19\bar{1}\bar{8}, 29\bar{2}\bar{10}$
	$1 \mapsto 1, 2 \mapsto 1, 3 \mapsto 3, 4 \mapsto 1, 5 \mapsto 1, 6 \mapsto 1, 7 \mapsto 2, 8 \mapsto 2, 9 \mapsto 21\bar{2}, 10 \mapsto 2$

Table 2: Surjective homomorphism to 4_1

K	relators
	surjective homomorphism to 4_1
8_{18}	$41\bar{4}\bar{2}, 53\bar{5}\bar{2}, 63\bar{6}\bar{4}, 75\bar{7}\bar{4}, 85\bar{8}\bar{6}, 17\bar{1}\bar{6}, 27\bar{2}\bar{8}$
	$1 \mapsto 2, 2 \mapsto 3, 3 \mapsto 4, 4 \mapsto 1, 5 \mapsto 2, 6 \mapsto 3, 7 \mapsto 4, 8 \mapsto 1$
9_{37}	$81\bar{8}\bar{2}, 72\bar{8}\bar{3}, 94\bar{9}\bar{3}, 34\bar{3}\bar{5}, 16\bar{1}\bar{5}, 56\bar{5}\bar{7}, 27\bar{2}\bar{8}, 49\bar{4}\bar{8}$
	$1 \mapsto 2, 2 \mapsto 3, 3 \mapsto 14\bar{1}, 4 \mapsto 3, 5 \mapsto 1, 6 \mapsto \bar{1}41, 7 \mapsto 4, 8 \mapsto 1, 9 \mapsto 4$
9_{40}	$81\bar{8}\bar{2}, 73\bar{7}\bar{2}, 64\bar{6}\bar{4}, 24\bar{2}\bar{5}, 16\bar{1}\bar{5}, 96\bar{9}\bar{7}, 57\bar{5}\bar{8}, 49\bar{4}\bar{8}$
	$1 \mapsto 1, 2 \mapsto 1, 3 \mapsto 2, 4 \mapsto 2, 5 \mapsto 3, 6 \mapsto 2, 7 \mapsto 4, 8 \mapsto 1, 9 \mapsto \bar{1}41$
10_{58}	$81\bar{8}\bar{2}, 42\bar{4}\bar{3}, 104\bar{10}\bar{3}, 24\bar{2}\bar{5}, 75\bar{7}\bar{6}, 97\bar{9}\bar{6}, 57\bar{5}\bar{8}, 18\bar{1}\bar{9}, 610\bar{6}\bar{9}$
	$1 \mapsto 1, 2 \mapsto 1, 3 \mapsto 21\bar{2}, 4 \mapsto 2, 5 \mapsto 3, 6 \mapsto 14\bar{1}, 7 \mapsto 4, 8 \mapsto 1,$ $9 \mapsto 1, 10 \mapsto 3$
10_{59}	$52\bar{5}\bar{1}, 93\bar{9}\bar{2}, 63\bar{6}\bar{4}, 15\bar{1}\bar{4}, 76\bar{7}\bar{5}, 36\bar{3}\bar{7}, 48\bar{4}\bar{7}, 108\bar{10}\bar{9}, 210\bar{2}\bar{9}$
	$1 \mapsto 1, 2 \mapsto 1, 3 \mapsto 4, 4 \mapsto 1, 5 \mapsto 1, 6 \mapsto 3, 7 \mapsto 14\bar{1}, 8 \mapsto 4, 9 \mapsto 3, 10 \mapsto 2$
10_{60}	$51\bar{5}\bar{2}, 13\bar{1}\bar{2}, 93\bar{9}\bar{4}, 24\bar{2}\bar{5}, 36\bar{3}\bar{5}, 106\bar{10}\bar{7}, 68\bar{6}\bar{7}, 48\bar{4}\bar{9}, 79\bar{7}\bar{10}$
	$1 \mapsto 4, 2 \mapsto 1, 3 \mapsto 2, 4 \mapsto 2, 5 \mapsto 3, 6 \mapsto 4, 7 \mapsto 1, 8 \mapsto 2, 9 \mapsto 2, 10 \mapsto 3$
10_{122}	$91\bar{9}\bar{2}, 83\bar{8}\bar{2}, 104\bar{10}\bar{3}, 14\bar{1}\bar{5}, 26\bar{2}\bar{5}, 46\bar{4}\bar{7}, 38\bar{3}\bar{7}, 59\bar{5}\bar{8}, 69\bar{6}\bar{10}$
	$1 \mapsto 2, 2 \mapsto 1, 3 \mapsto 1, 4 \mapsto 4, 5 \mapsto 3, 6 \mapsto 2, 7 \mapsto 1, 8 \mapsto 1, 9 \mapsto 4, 10 \mapsto 3$
10_{136}	$52\bar{5}\bar{1}, 63\bar{6}\bar{2}, 24\bar{2}\bar{6}, 94\bar{9}\bar{5}, 86\bar{8}\bar{5}, 37\bar{3}\bar{6}, 107\bar{10}\bar{8}, 19\bar{1}\bar{8}, 29\bar{2}\bar{10}$
	$1 \mapsto 21\bar{2}, 2 \mapsto 1, 3 \mapsto 4, 4 \mapsto \bar{1}41, 5 \mapsto 2, 6 \mapsto 3, 7 \mapsto \bar{3}23, 8 \mapsto 2\bar{3}23\bar{2},$ $9 \mapsto \bar{1}21, 10 \mapsto 2$
10_{137}	$51\bar{5}\bar{2}, 13\bar{1}\bar{5}, 103\bar{10}\bar{4}, 24\bar{2}\bar{5}, 36\bar{3}\bar{5}, 86\bar{8}\bar{7}, 108\bar{10}\bar{7}, 18\bar{1}\bar{9}, 49\bar{4}\bar{10}$
	$1 \mapsto 2, 2 \mapsto 23\bar{2}, 3 \mapsto 3, 4 \mapsto 3, 5 \mapsto 21\bar{2}, 6 \mapsto 2, 7 \mapsto 1, 8 \mapsto 4,$ $9 \mapsto 3, 10 \mapsto 3$
10_{138}	$51\bar{5}\bar{2}, 13\bar{1}\bar{2}, 84\bar{8}\bar{3}, 24\bar{2}\bar{5}, 36\bar{3}\bar{5}, 106\bar{10}\bar{7}, 68\bar{6}\bar{7}, 39\bar{3}\bar{8}, 79\bar{7}\bar{10}$
	$1 \mapsto 4, 2 \mapsto 1, 3 \mapsto 2, 4 \mapsto 2, 5 \mapsto 3, 6 \mapsto 4, 7 \mapsto 1, 8 \mapsto 2, 9 \mapsto 2, 10 \mapsto 3$

Table 3: Surjective homomorphism to 5_2

K	relators
	surjective homomorphism to 5_2
10_{74}	$61\bar{6}\bar{2}, 42\bar{4}\bar{3}, 84\bar{8}\bar{3}, 104\bar{10}\bar{5}, 95\bar{9}\bar{6}, 16\bar{1}\bar{7}, 27\bar{2}\bar{8}, 39\bar{3}\bar{8}, 59\bar{5}\bar{10}$
	$1 \mapsto \bar{2}12, 2 \mapsto 2, 3 \mapsto 12\bar{1}, 4 \mapsto 1, 5 \mapsto 12\bar{1}, 6 \mapsto 4\bar{1}2, 7 \mapsto \bar{2}52, 8 \mapsto 5,$ $9 \mapsto 13\bar{1}, 10 \mapsto 5$
10_{120}	$51\bar{5}\bar{2}, 92\bar{9}\bar{3}, 13\bar{1}\bar{4}, 74\bar{7}\bar{5}, 35\bar{3}\bar{6}, 106\bar{10}\bar{7}, 47\bar{4}\bar{8}, 68\bar{6}\bar{9}, 29\bar{2}\bar{10}$
	$1 \mapsto 3, 2 \mapsto 4, 3 \mapsto 5, 4 \mapsto 1, 5 \mapsto 2, 6 \mapsto 3, 7 \mapsto 4, 8 \mapsto 5, 9 \mapsto 1, 10 \mapsto 2$
10_{122}	$91\bar{9}\bar{2}, 83\bar{8}\bar{2}, 104\bar{10}\bar{3}, 14\bar{1}\bar{5}, 26\bar{2}\bar{5}, 46\bar{4}\bar{7}, 38\bar{3}\bar{7}, 59\bar{5}\bar{8}, 69\bar{6}\bar{10}$
	$1 \mapsto 2, 2 \mapsto 2, 3 \mapsto 1, 4 \mapsto 5, 5 \mapsto 25\bar{2}, 6 \mapsto 5, 7 \mapsto 5, 8 \mapsto 4, 9 \mapsto 2, 10 \mapsto 3$

Table 4: Non-existence of surjective homomorhpsim

3_1	$8_{11}(5), 9_{29}(3), 9_{38}(3), 10_{59}(3), 10_{113}(3), 10_{122}(5), 10_{136}(3), 10_{147}(5)$
4_1	$8_{21}(3), 9_{12}(3), 9_{24}(3), 9_{39}(3)$
5_1	$10_{21}(5), 10_{62}(5), 10_{100}(5), 10_{132}(5)$
5_2	$9_{12}(5), 10_{65}(17), 10_{67}(5), 10_{77}(7), 10_{95}(5), 10_{111}(7)$
6_1	$8_{11}(7), 9_{37}(7), 9_{46}(11), 10_{21}(7), 10_{67}(7), 10_{74}(11), 10_{87}(7), 10_{98}(7), 10_{147}(11)$
6_2	$10_{111}(7), 10_{123}(7)$
6_3	$10_{95}(5), 10_{100}(5), 10_{159}(5)$
7_2	$8_{15}(3), 9_{39}(3)$
7_3	$9_{16}(3)$
7_4	$9_2(5), 9_{23}(7), 10_{120}(7)$
7_5	$10_{130}(11)$
8_1	$10_{58}(5), 10_{144}(7)$
8_3	$10_1(7)$
8_4	$10_{76}(3)$
8_5	$10_{82}(2), 10_{141}(3)$
8_6	$10_{32}(5)$
8_8	$10_{40}(7), 10_{103}(7), 10_{129}(5)$
8_9	$10_{106}(7), 10_{155}(7)$
8_{10}	$10_{99}(7), 10_{143}(7)$
8_{11}	$10_{87}(2), 10_{98}(7), 10_{147}(7)$
8_{13}	$10_{84}(3)$
8_{14}	$9_8(11), 10_{114}(11), 10_{131}(11)$
8_{16}	$10_{156}(5)$
8_{18}	$9_{24}(2)$
8_{20}	$8_{10}(5), 8_{18}(2), 9_{24}(5),$ $10_{62}(5), 10_{65}(5), 10_{77}(5), 10_{82}(5), 10_{87}(5), 10_{98}(5), 10_{99}(5), 10_{140}(5), 10_{143}(5)$
8_{21}	$8_{18}(2), 9_{24}(5), 9_{40}(3), 10_{59}(2), 10_{122}(2), 10_{136}(2)$
9_2	$7_4(11), 9_{23}(11), 10_{120}(11)$
9_8	$8_{14}(5), 10_{114}(5), 10_{131}(5)$
9_{12}	$10_{122}(3)$
9_{14}	$10_{113}(3)$
9_{15}	$10_{166}(7)$
9_{20}	$10_{149}(5)$
9_{24}	$8_{18}(2)$
9_{28}	$9_{29}(2), 10_{164}(2)$
9_{29}	$9_{28}(2), 10_{164}(3)$
9_{38}	$10_{63}(3)$
9_{40}	$10_{59}(2)$
9_{42}	$8_5(3), 10_{82}(3), 10_{138}(3), 10_{141}(3)$
9_{44}	$9_{28}(3), 9_{29}(3), 10_{60}(3), 10_{164}(3)$
9_{45}	$10_{78}(5)$
9_{46}	$6_1(5), 8_{11}(5), 9_{37}(5), 10_{21}(5), 10_{67}(5), 10_{74}(5), 10_{87}(7), 10_{98}(7), 10_{147}(5)$
9_{49}	$10_{66}(3)$
10_1	$8_3(7)$
	Continued on next page

Table 4 Continued from previous page

10_{10}	$10_{165}(7)$
10_{12}	$10_{54}(7)$
10_{18}	$10_{24}(7)$
10_{20}	$10_{163}(5)$
10_{23}	$10_{52}(7)$
10_{24}	$10_{18}(7)$
10_{25}	$10_{56}(5)$
10_{28}	$10_{37}(3)$
10_{31}	$10_{68}(5)$
10_{34}	$10_{135}(5)$
10_{37}	$10_{28}(3)$
10_{40}	$10_{103}(3)$
10_{42}	$10_{75}(2)$
10_{52}	$10_{23}(5)$
10_{54}	$10_{12}(5)$
10_{56}	$10_{25}(5)$
10_{59}	$9_{40}(2)$
10_{63}	$9_{38}(2)$
10_{65}	$10_{77}(3)$
10_{67}	$10_{74}(5)$
10_{68}	$10_{31}(5)$
10_{74}	$10_{67}(2)$
10_{75}	$10_{42}(2)$
10_{77}	$10_{65}(5)$
10_{87}	$10_{98}(5)$
10_{98}	$10_{87}(2)$
10_{103}	$10_{40}(3)$
10_{125}	$10_{5}(5)$
10_{127}	$10_{112}(5), 10_{150}(5)$
10_{129}	$8_{8}(5), 10_{40}(5), 10_{103}(5)$
10_{130}	$7_{5}(5)$
10_{131}	$8_{14}(5), 9_{8}(5), 10_{114}(7)$
10_{132}	$5_{1}(5), 10_{21}(5), 10_{62}(5), 10_{100}(5)$
10_{135}	$10_{34}(5)$
10_{136}	$8_{18}(2), 8_{21}(2), 9_{24}(3), 9_{40}(5), 10_{59}(3), 10_{122}(3)$
10_{137}	$9_{40}(5), 10_{59}(7)$
10_{140}	$8_{10}(3), 8_{18}(2), 8_{20}(3), 9_{24}(3), 10_{62}(3), 10_{65}(3),$ $10_{77}(3), 10_{82}(3), 10_{87}(3), 10_{98}(3), 10_{99}(3), 10_{143}(3)$
10_{141}	$8_{5}(3), 10_{82}(2)$
10_{143}	$8_{10}(5), 10_{99}(5)$
10_{147}	$8_{11}(5), 10_{87}(2), 10_{98}(5)$
10_{149}	$9_{20}(5)$
10_{150}	$10_{112}(5), 10_{127}(5)$
10_{155}	$8_{9}(5), 10_{106}(5)$
10_{156}	$8_{16}(5)$

Continued on next page

Table 4 Continued from previous page

10_{163}	$10_{20}(5)$
10_{164}	$9_{28}(2), 9_{29}(3)$
10_{165}	$10_{10}(2)$
10_{166}	$9_{15}(5)$

Table 5: Twisted Alexander invariant

K	p	$\left(\Delta^{(n)}_{K,\rho}(t), \Delta^{(d)}_{K,\rho}(t)\right)$
3_1	3	$(0+1+1, 1+1), (0+2+1, 2+1), (2+0+1, 0+1),$ $(2+1+1, 1+1), (2+2+1, 2+1)$
	5	$(0+1+1, 1+1), (0+4+1, 4+1), (2+0+1, 0+1),$ $(2+1+1, 1+1), (2+2+1, 2+1), (2+3+1, 3+1),$ $(2+4+1, 4+1), (3+2+1, 2+1), (3+3+1, 3+1),$ $(4+0+1, 0+1)$
4_1	3	$(1+0+1, 0+1), (1+0+1, 1+1), (1+0+1, 2+1),$ $(2+0+1, 1+1), (2+0+1, 2+1)$
5_1	5	$(0+1+3+3+1, 3+1), (0+4+3+2+1, 2+1),$ $(1+0+4+0+1, 0+1), (1+1+4+2+1, 2+1),$ $(1+2+4+4+1, 4+1), (1+3+4+1+1, 1+1),$ $(1+4+4+3+1, 3+1), (4+1+0+4+1, 4+1),$ $(4+4+0+1+1, 1+1)$
5_2	5	$(0+1+4, 1+1), (0+4+4, 4+1), (1+0+4, 0+1),$ $(2+2+4, 2+1), (2+3+4, 3+1), (3+1, 3+1),$ $(3+4, 2+1)$
	7	$(1+0+4, 0+1), (2+2, 1+1), (2+3+4, 4+1),$ $(2+4+4, 3+1), (2+5, 6+1), (3+1+1, 5+1),$ $(3+2+4, 5+1), (3+5+4, 2+1), (3+6+1, 2+1),$ $(5+1+4, 6+1), (5+6+4, 1+1)$
	17	$(0+12+4, 2+1), (0+5+4, 15+1), (1+0+4, 0+1),$ $(10+8+4, 7+1), (10+9+4, 10+1), (1+11+11, 10+1),$ $(14+0+15, 14+1), (14+0+15, 3+1), (14+10+4, 13+1),$ $(14+7+4, 4+1), (14+8+5, 6+1), (14+9+5, 11+1),$ $(1+6+11, 7+1), (16+13+4, 5+1), (16+4+4, 12+1),$ $(16+8+7, 16+1), (16+9+7, 1+1), (2+14+4, 8+1),$ $(2+3+4, 9+1), (3+1+4, 3+1), (3+16+4, 14+1),$ $(3+8+1, 14+1), (3+9+1, 3+1), (4+13+6, 8+1),$ $(4+4+6, 9+1), (5+11+4, 16+1), (5+14+13, 10+1),$ $(5+3+13, 7+1), (5+6+4, 1+1), (7+14+8, 15+1),$ $(7+3+8, 2+1), (9+15+4, 11+1), (9+2+4, 6+1)$
		Continued on next page

Table 5 Continued from previous page

K	p	$\left(\Delta^{(n)}_{K,\rho}(t), \Delta^{(d)}_{K,\rho}(t)\right)$
6_1	7	$(0+0+3,3+1),(0+0+3,4+1),(0+3+4,1+1),$ $(0+4+4,6+1),(2+0+1,0+1),(3+0+4,0+1),$ $(3+1,3+1),(3+6,4+1),(4+2+4,3+1),$ $(4+5+4,4+1),(5+1+4,5+1),(5+6+4,2+1)$
	11	$(0+2+4,9+1),(0+9+4,2+1),(10+10+4,1+1),$ $(10+1+4,10+1),(2+0+1,0+1),(4+4+4,7+1),$ $(4+7+4,4+1),(6+0+4,0+1),(7+5+4,6+1),$ $(7+5+7,2+1),(7+6+4,5+1),(7+6+7,9+1),$ $(9+2,10+1),(9+3+4,8+1),(9+8+4,3+1),$ $(9+9,1+1)$
6_2	7	$(0+0+3+0+1,0+1),(0+3+2+5+1,4+1),$ $(0+4+2+2+1,3+1),(1+3+1+6+1,2+1),$ $(1+4+1+1+1,5+1),(4+2+5+6+1,2+1),$ $(4+5+5+1+1,5+1),(6+3+6+3+1,1+1),$ $(6+4+6+4+1,6+1)$
6_3	5	$(0+1+4+1+1,2+1),(0+1+4+2+1,4+1),$ $(0+4+4+3+1,1+1),(0+4+4+4+1,3+1),$ $(4+0+4+0+1,0+1),(4+2+2+3+1,1+1),$ $(4+3+2+2+1,4+1)$
7_2	3	$(1,0+1),(1+0+1,1+1),(1+0+1,2+1),$ $(1,1+1),(1,2+1)$
7_3	3	$(2+0+0+0+1,0+1),(2+0+0+0+1,1+1),$ $(2+0+0+0+1,2+1),(2+1+1+1+2,1+1),$ $(2+2+1+2+2,2+1)$
7_4	5	$(1+1,1+1),(1+1+1,2+1),(1+4+1,3+1),$ $(1+4,4+1),(2+0+1,0+1),(3+2+1,4+1),$ $(3+3+1,1+1)$
	7	$(0+0+1,3+1),(0+0+1,4+1),(0+0+2,3+1),$ $(0+0+2,4+1),(2+0+1,0+1),(3+0+2,0+1),$ $(3+1,3+1),(3+1+4,6+1),(3+6,4+1),$ $(3+6+4,1+1),(4+0+2,2+1),(4+0+2,5+1),$ $(5+0+2,1+1),(5+0+2,6+1),(6+2+2,3+1),$ $(6+2+2,5+1),(6+5+2,2+1),(6+5+2,4+1)$
7_5	11	$(10+10+3+6+4,9+1),(10+1+3+5+4,2+1),$ $(1+0+7+0+4,0+1),(2+10+2+1+4,7+1),$ $(2+1+2+10+4,4+1),(3+3+0+4+10,5+1),$ $(3+8+0+7+10,6+1),(4+10+4+4+4,6+1),$ $(4+1+4+7+4,5+1),(5+4+6+8+4,1+1),$ $(5+7+6+3+4,10+1),(6+5+9+9+4,8+1),$ $(6+6+9+2+4,3+1),(8+5+1+6+6,5+1),$ $(8+6+1+5+6,6+1)$

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Table 5 Continued from previous page

K	p	$\left(\Delta^{(n)}_{K,\rho}(t), \Delta^{(d)}_{K,\rho}(t)\right)$
8_1	5	$(0+1+4, 1+1), (0+4+4, 4+1), (1+0+4, 0+1),$ $(2+1+1, 1+1), (2+2+4, 2+1), (2+3+4, 3+1),$ $(2+4+1, 4+1), (3+0+1, 2+1), (3+0+1, 3+1)$
	7	$(0+0+2, 3+1), (0+0+2, 4+1), (2+1+1, 1+1),$ $(2+3+2, 2+1), (2+4+2, 5+1), (2+6+1, 6+1),$ $(3+0+2, 0+1), (4+0+2, 2+1), (4+0+2, 5+1),$ $(5+0+2, 1+1), (5+0+2, 6+1)$
8_3	7	$(0+0+2, 0+1), (0+3+4, 1+1), (0+4+4, 6+1),$ $(1+2+1, 4+1), (1+2+2, 2+1), (1+5+1, 3+1),$ $(1+5+2, 5+1), (2+1+2, 1+1), (2+6+2, 6+1),$ $(4+3+2, 3+1), (4+3+2, 5+1), (4+4+2, 2+1),$ $(4+4+2, 4+1)$
8_4	3	$(1+0+2+0+1, 0+1), (2+1+0+2+1, 2+1),$ $(2+1+1+1+2, 1+1), (2+2+0+1+1, 1+1),$ $(2+2+1+2+2, 2+1)$
8_5	2	$(0+0+1+0+1+0+1, 0+1), (0+1+1+0+1+1+1, 1+1),$ $(1+0+0+0+1+0+1, 0+1)$
	3	$(0+0+0+0+1+0+1, 0+1), (1+1+1+0+0+0+1, 2+1),$ $(1+2+1+0+0+0+1, 1+1), (2+1+0+1+2+0+1, 2+1),$ $(2+2+0+2+2+0+1, 1+1)$
8_6	5	$(0+0+3+0+4, 0+1), (2+2+1, 2+1), (2+3+1, 3+1),$ $(3+1+4+1+3, 4+1), (3+4+4+4+3, 1+1),$ $(4+2+2+3+4, 4+1), (4+2+4+1+4, 3+1),$ $(4+3+2+2+4, 1+1), (4+3+4+4+4, 2+1)$
8_8	5	$(1+0+3+2+4, 1+1), (1+0+3+3+4, 4+1),$ $(1+1+3, 2+1), (1+2+2+4+4, 2+1),$ $(1+3+2+1+4, 3+1), (1+4+3, 3+1),$ $(2+0+0+0+4, 0+1)$
	7	$(0+1+2+4+4, 5+1), (0+6+2+3+4, 2+1),$ $(1+0+5+1+6, 2+1), (1+0+5+6+6, 5+1),$ $(3+0+0+0+4, 0+1), (3+0+1+1+4, 3+1),$ $(3+0+1+6+4, 4+1), (6+2+4+5+4, 1+1),$ $(6+5+4+2+4, 6+1)$
8_9	7	$(0+1+0+2+5+6+1, 2+1), (0+6+0+5+5+1+1, 5+1),$ $(1+0+0+0+6+0+1, 0+1), (3+0+2+3+2+2+1, 3+1),$ $(3+0+2+4+2+5+1, 4+1), (3+1+6+1+4+3+1, 1+1),$ $(3+2+1+3+2+2+1, 3+1), (3+5+1+4+2+5+1, 4+1),$ $(3+6+6+6+4+4+1, 6+1), (6+2+3+5+5+5+1, 4+1),$ $(6+5+3+2+5+2+1, 3+1)$

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Table 5 Continued from previous page

K	p	$\left(\Delta^{(n)}_{K,\rho}(t), \Delta^{(d)}_{K,\rho}(t)\right)$
8_{10}	7	$(0+0+6+0+4+0+1, 0+1), (1+0+3+2+0+4+1, 6+1),$ $(1+0+3+5+0+3+1, 1+1), (1+0+6+1+0+6+1, 2+1),$ $(1+0+6+6+0+1+1, 5+1), (1+2+0+4+4+5+1, 4+1),$ $(1+2+1+0+5+1+1, 5+1), (1+2+2+4+6+4+1, 6+1),$ $(1+5+0+3+4+2+1, 3+1), (1+5+1+0+5+6+1, 2+1),$ $(1+5+2+3+6+3+1, 1+1), (2+0+3+0+3+0+1, 0+1),$ $(2+1+1+0+1+0+1, 0+1), (2+2+1+1+6+2+1, 3+1),$ $(2+5+1+6+6+5+1, 4+1), (2+6+1+0+1+0+1, 0+1),$ $(3+0+3+2+2+5+1, 4+1), (3+0+3+5+2+2+1, 3+1),$ $(4+1+5+4+4+5+1, 4+1), (4+6+5+3+4+2+1, 3+1),$ $(5+2+3+0+5+6+1, 2+1), (5+5+3+0+5+1+1, 5+1),$ $(6+2+2+4+3+3+1, 1+1), (6+5+2+3+3+4+1, 6+1)$
8_{11}	2	$(0+0+1+0+1, 0+1), (0+1+1, 1+1), (1+0+1, 0+1)$
	7	$(0+0+1+0+1, 0+1), (0+0+3+0+4, 1+1),$ $(0+0+3+0+4, 6+1), (3+0+0+0+4, 3+1),$ $(3+0+0+0+4, 4+1), (4+1+1+3+6, 4+1),$ $(4+2+1+5+3, 3+1), (4+5+1+2+3, 4+1),$ $(4+6+1+4+6, 3+1), (5+0+1+0+4, 2+1),$ $(5+0+1+0+4, 5+1), (5+0+6+0+4, 0+1),$ $(6+1+5+1+2, 1+1), (6+6+5+6+2, 6+1)$
8_{13}	3	$(0+0+2+1+2, 2+1), (0+0+2+2+2, 1+1),$ $(1+0+2+0+1, 0+1), (2+1+0+2+1, 1+1),$ $(2+2+0+1+1, 2+1)$
8_{14}	11	$(0+4+8+1+8, 2+1), (0+4+9+2+4, 7+1),$ $(0+7+8+10+8, 9+1), (0+7+9+9+4, 4+1),$ $(1+0+9+0+4, 0+1), (2+3+3+2+2, 9+1),$ $(2+8+3+9+2, 2+1), (4+10+9+10+4, 2+1),$ $(4+1+9+1+4, 9+1), (5+0+9+3+4, 5+1),$ $(5+0+9+8+4, 6+1), (5+3+2+1+5, 7+1),$ $(5+8+2+10+5, 4+1), (8+0+5+10+5, 4+1),$ $(8+0+5+1+5, 7+1), (9+10+9+6+4, 10+1),$ $(9+1+9+5+4, 1+1), (9+2+9+4+4, 3+1),$ $(9+9+9+7+4, 8+1)$
8_{16}	5	$(1+0+4+0+0+1+1, 4+1), (1+0+4+0+0+4+1, 1+1),$ $(3+0+3+0+0+0+1, 0+1), (3+1+2+2+2+3+1, 2+1),$ $(3+2+0+4+3+4+1, 1+1), (3+3+0+1+3+1+1, 4+1),$ $(3+4+2+3+2+2+1, 3+1)$
8_{18}	2	$(0+0+0+0+1+0+1, 0+1), (0+1+1+0+1+1+1, 1+1),$ $(1+0+0+0+1+0+1, 0+1)$

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Table 5 Continued from previous page

K	p	$\left(\Delta^{(n)}_{K,\rho}(t), \Delta^{(d)}_{K,\rho}(t)\right)$
8_{20}	2	$(0+0+0+0+1, 0+1), (0+0+1+0+1, 1+1),$ $(1+0+0+0+1, 0+1)$
	5	$(1+0+4+0+1, 0+1), (2+0+0+2+1, 1+1),$ $(2+0+0+3+1, 4+1), (3+0+3+0+1, 0+1),$ $(3+1+0+2+1, 1+1), (3+4+0+3+1, 4+1),$ $(4+1+0+4+1, 2+1), (4+2+1+2+1, 1+1),$ $(4+2+3+4+1, 2+1), (4+3+1+3+1, 4+1),$ $(4+3+3+1+1, 3+1), (4+4+0+1+1, 3+1)$
8_{21}	2	$(0+0+0+0+1, 0+1), (0+0+1+0+1, 1+1),$ $(1+0+0+0+1, 0+1)$
	3	$(1+0+0+0+1, 0+1), (2+0+2+1+1, 1+1),$ $(2+0+2+1+1, 2+1), (2+0+2+2+1, 1+1),$ $(2+0+2+2+1, 2+1)$
	5	$(0+0+1+0+1, 0+1), (1+2+4+4+1, 4+1),$ $(1+3+4+1+1, 1+1), (2+1+2+0+1, 3+1),$ $(2+4+2+0+1, 2+1), (3+0+0+0+1, 0+1),$ $(3+1+0+2+1, 1+1), (3+2+1+4+1, 1+1),$ $(3+3+1+1+1, 4+1), (3+4+0+3+1, 4+1),$ $(4+2+1+2+1, 3+1), (4+3+1+3+1, 2+1)$
9_2	11	$(0+5+5, 10+1), (0+6+5, 1+1), (10+3+5, 6+1),$ $(10+8+5, 5+1), (2+0+1, 0+1), (3+10, 8+1),$ $(3+1, 3+1), (4+10+5, 9+1), (4+1+5, 2+1),$ $(6+0+5, 0+1), (7+3+10, 9+1), (7+4+5, 8+1),$ $(7+7+5, 3+1), (7+8+10, 2+1), (9+2+5, 4+1),$ $(9+9+5, 7+1)$
9_8	5	$(1+0+0+0+4, 0+1), (1+2+2+4+3, 4+1),$ $(1+3+2+1+3, 1+1), (2+2+3+3+4, 3+1),$ $(2+3+3+2+4, 2+1), (3+1+2+1+4, 1+1),$ $(3+4+2+4+4, 4+1)$
9_{12}	3	$(0+0+1+0+1, 1+1), (0+0+1+0+1, 2+1),$ $(0+0+2+0+1, 0+1), (0+0+2+1+2, 2+1),$ $(0+0+2+2+2, 1+1)$
9_{14}	3	$(1+1+1+1+2, 2+1), (1+2+1+2+2, 1+1),$ $(2+0+0+0+1, 0+1), (2+0+0+0+1, 1+1),$ $(2+0+0+0+1, 2+1)$
9_{15}	7	$(0+0+1+0+1, 0+1), (0+2+2+3+4, 4+1),$ $(0+5+2+4+4, 3+1), (2+2+0+1+4, 6+1),$ $(2+5+0+6+4, 1+1), (3+0+5+2+1, 6+1),$ $(3+0+5+5+1, 1+1), (5+0+5+0+4, 0+1),$ $(5+2+4+4+3, 4+1), (5+5+4+3+3, 3+1),$ $(6+1+2+2+5, 4+1), (6+3+6+2+4, 5+1),$ $(6+4+6+5+4, 2+1), (6+6+2+5+5, 3+1)$

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Table 5 Continued from previous page

K	p	$\left(\Delta^{(n)}_{K,\rho}(t), \Delta^{(d)}_{K,\rho}(t)\right)$
9_{20}	5	$(0+2+4+3+3+0+1, 3+1), (0+3+4+2+3+0+1, 2+1),$ $(2+0+4+0+2+0+1, 0+1), (2+1+2+2+1+0+1, 4+1),$ $(2+2+1+2+1+0+1, 2+1), (2+3+1+3+1+0+1, 3+1),$ $(2+4+2+3+1+0+1, 1+1)$
9_{24}	2	$(0+0+1+0+1+0+1, 0+1), (0+1+1+0+1+1+1, 1+1),$ $(1+0+0+0+1+0+1, 0+1)$
9_{28}	2	$(0+0+1+0+1+0+1, 0+1), (0+1+1+0+1+1+1, 1+1),$ $(1+0+0+0+1+0+1, 0+1)$
9_{29}	2	$(0+0+0+0+1+0+1, 0+1), (0+1+1+0+1+1+1, 1+1),$ $(1+0+0+0+1+0+1, 0+1)$
	3	$(0+0+0+0+1+0+1, 0+1), (1+1+0+0+1+1+1, 2+1),$ $(1+2+0+0+1+2+1, 1+1), (2+1+2+2+0+1+1, 2+1),$ $(2+2+2+1+0+2+1, 1+1)$
9_{38}	3	$(0+0+2+0+1, 1+1), (0+0+2+0+1, 2+1),$ $(1+0+0+0+1, 0+1), (2+0+2+1+1, 1+1),$ $(2+0+2+2+1, 2+1)$
9_{40}	2	$(0+0+1+0+1+0+1, 0+1), (0+1+1+0+1+1+1, 1+1),$ $(1+0+0+0+1+0+1, 0+1)$
9_{42}	3	$(1+0+2+0+1, 0+1), (2+1+0+1+1, 2+1),$ $(2+1+0+2+1, 1+1), (2+2+0+1+1, 2+1),$ $(2+2+0+2+1, 1+1)$
9_{44}	3	$(1+0+2+0+1, 0+1), (2+1+0+1+1, 2+1),$ $(2+1+0+2+1, 2+1), (2+2+0+1+1, 1+1),$ $(2+2+0+2+1, 1+1)$
9_{45}	5	$(0+0+4+2+1, 2+1), (0+0+4+3+1, 3+1),$ $(1+0+3+0+1, 0+1), (2+1+4+4+1, 4+1),$ $(2+4+4+1+1, 1+1), (4+2+2+1+1, 1+1),$ $(4+2+3+1+1, 0+1), (4+3+2+4+1, 4+1),$ $(4+3+3+4+1, 0+1)$
9_{46}	5	$(1+0+4, 1+1), (1+0+4, 4+1), (1+1+2, 4+1),$ $(1+4+2, 1+1), (2+0+1, 0+1), (2+0+4, 0+1),$ $(2+1+1, 1+1), (2+2+2, 3+1), (2+3+2, 2+1),$ $(2+4+1, 4+1), (3+0+4, 0+1), (3+0+4, 2+1),$ $(3+0+4, 3+1)$
	7	$(0+0+3, 3+1), (0+0+3, 4+1), (0+3+4, 1+1),$ $(0+4+4, 6+1), (1+0+4, 0+1), (1+2+2, 4+1),$ $(1+5+2, 3+1), (2+0+1, 0+1), (2+2+1, 2+1),$ $(2+3+1, 3+1), (2+4+1, 4+1), (2+5+1, 5+1),$ $(3+0+4, 0+1), (4+0+2, 0+1), (4+2+4, 3+1),$ $(4+5+4, 4+1), (5+1+4, 5+1), (5+6+4, 2+1)$
9_{49}	3	$(1+0+0+1, 1+1), (1+0+0+2, 2+1), (1, 0+1),$ $(1, 1+1), (1, 2+1)$
		Continued on next page

Table 5 Continued from previous page

K	p	$\left(\Delta^{(n)}_{K,\rho}(t), \Delta^{(d)}_{K,\rho}(t)\right)$
10_1	7	$(0+0+2, 0+1), (1+2+2, 2+1), (1+5+2, 5+1),$ $(2+1+2, 1+1), (2+6+2, 6+1), (3+1+1, 5+1),$ $(3+6+1, 2+1), (4+3+2, 3+1), (4+4+2, 4+1)$
10_{10}	7	$(2+0+1, 0+1), (2+0+5+0+2, 0+1),$ $(3+1+6+4+2, 5+1), (3+2+4+6+4, 3+1),$ $(3+5+4+1+4, 4+1), (3+6+6+3+2, 2+1),$ $(4+2+5+0+2, 2+1), (4+5+5+0+2, 5+1),$ $(5+0+2+1+2, 3+1), (5+0+2+6+2, 4+1),$ $(5+2+0+5+2, 1+1), (5+5+0+2+2, 6+1)$
10_{12}	7	$(0+0+3+0+5+3+6, 6+1), (0+0+3+0+5+4+6, 1+1),$ $(0+2+0+2+2+5+4, 1+1), (0+2+4+2+1+1+4, 3+1),$ $(0+5+0+5+2+2+4, 6+1), (0+5+4+5+1+6+4, 4+1),$ $(2+0+6+3+6+3+4, 2+1), (2+0+6+4+6+4+4, 5+1),$ $(5+3+1+1+0+5+5, 5+1), (5+4+1+6+0+2+5, 2+1),$ $(6+0+1+0+3+0+4, 0+1), (6+2+3+5+5+5+1, 4+1),$ $(6+2+5+6+6+1+3, 4+1), (6+5+3+2+5+2+1, 3+1),$ $(6+5+5+1+6+6+3, 3+1)$
10_{18}	7	$(0+0+0+0+2, 3+1), (0+0+0+0+2, 4+1),$ $(1+0+2+0+2, 0+1), (1+0+5+0+2, 2+1),$ $(1+0+5+0+2, 5+1), (1+0+5+2+6, 6+1),$ $(1+0+5+5+6, 1+1), (2+0+1+0+2, 1+1),$ $(2+0+1+0+2, 6+1)$
10_{20}	5	$(1+0+3+2+4, 1+1), (1+0+3+3+4, 4+1),$ $(1+2+0+4+1, 3+1), (1+2+2+4+4, 2+1),$ $(1+3+0+1+1, 2+1), (1+3+2+1+4, 3+1),$ $(2+0+0+0+4, 0+1)$
10_{23}	7	$(1+1+3+4+2+0+4, 6+1), (1+3+5+6+0+0+4, 4+1),$ $(1+4+5+1+0+0+4, 3+1), (1+6+3+3+2+0+4, 1+1),$ $(2+1+3+3+5+2+6, 5+1), (2+6+3+4+5+5+6, 2+1),$ $(4+2+4+3+3+0+4, 5+1), (4+2+6+3+0+5+5, 4+1),$ $(4+5+4+4+3+0+4, 2+1), (4+5+6+4+0+2+5, 3+1),$ $(5+0+4+0+4+0+4, 0+1)$
10_{24}	7	$(0+0+0+0+2, 3+1), (0+0+0+0+2, 4+1),$ $(0+0+2+3+2, 5+1), (0+0+2+4+2, 2+1),$ $(1+0+2+0+2, 0+1), (1+0+5+0+2, 2+1),$ $(1+0+5+0+2, 5+1), (2+0+1+0+2, 1+1),$ $(2+0+1+0+2, 6+1), (3+0+3+1+5, 5+1),$ $(3+0+3+6+5, 2+1), (5+1+3+5+5, 3+1),$ $(5+6+3+2+5, 4+1)$
10_{25}	5	$(2+0+0+0+3+0+4, 0+1), (2+0+2+1+1+4+4, 4+1),$ $(2+0+2+4+1+1+4, 1+1), (2+2+4+2+0+2+4, 2+1),$ $(2+3+4+3+0+3+4, 3+1), (4+2+0+3+1+3, 2+1),$ $(4+3+0+2+1+2, 3+1)$

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Table 5 Continued from previous page

K	p	$\left(\Delta^{(n)}_{K,\rho}(t), \Delta^{(d)}_{K,\rho}(t)\right)$
10_{28}	3	$(1+0+2+0+1, 0+1), (2+1+0+2+1, 2+1),$ $(2+1+1+1+2, 1+1), (2+2+0+1+1, 1+1),$ $(2+2+1+2+2, 2+1)$
10_{31}	5	$(0+0+1+0+1, 0+1), (0+0+4+2+1, 2+1),$ $(0+0+4+3+1, 3+1), (1+0+3+0+1, 0+1),$ $(1+2+0+4, 4+1), (1+3+0+1, 1+1),$ $(4+2+2+1+1, 1+1), (4+3+2+4+1, 4+1)$
10_{34}	5	$(0+0+3+0+4, 0+1), (0+2+2+1+1, 2+1),$ $(0+3+2+4+1, 3+1), (4+2+2+3+4, 4+1),$ $(4+2+4+1+4, 3+1), (4+3+2+2+4, 1+1),$ $(4+3+4+4+4, 2+1)$
10_{37}	3	$(1+0+2+0+1, 0+1), (2+1+0+2+1, 2+1),$ $(2+1+1+2+1, 2+1), (2+2+0+1+1, 1+1),$ $(2+2+1+1+1, 1+1)$
10_{40}	3	$(0+1+0+2+0+2+1, 2+1), (0+2+0+1+0+1+1, 1+1),$ $(1+0+0+0+2+0+1, 0+1), (1+0+2+0+2+1+2, 1+1),$ $(1+0+2+0+2+2+2, 2+1)$
10_{42}	2	$(0+0+0+0+0+0+1, 0+1), (1+0+1+0+1+0+1, 0+1),$ $(1+0+1+1+0+1+1, 1+1)$
10_{52}	5	$(0+2+3+1+3+4+4, 1+1), (0+3+3+4+3+1+4, 4+1),$ $(2+0+1+0+2+0+4, 0+1), (3+2+1+4+3+1, 3+1),$ $(3+3+1+1+3+4, 2+1), (4+0+1+2+1+3+4, 2+1),$ $(4+0+1+3+1+2+4, 3+1)$
10_{54}	5	$(0+0+3+0+1+0+4, 0+1), (1+1+2+1+1+1+4, 3+1),$ $(1+4+2+4+1+4+4, 2+1), (3+2+2+1+1, 3+1),$ $(3+2+4+1+1+2+4, 1+1), (3+3+2+4+1, 2+1),$ $(3+3+4+4+1+3+4, 4+1)$
10_{56}	5	$(2+0+0+0+3+0+4, 0+1), (2+0+2+1+1+4+4, 4+1),$ $(2+0+2+4+1+1+4, 1+1), (2+1+0+2+4+0+2, 1+1),$ $(2+2+4+2+0+2+4, 2+1), (2+3+4+3+0+3+4, 3+1),$ $(2+4+0+3+4+0+2, 4+1), (4+1+1+3+4+3, 2+1),$ $(4+4+1+2+4+2, 3+1)$
10_{59}	2	$(0+0+0+0+0+0+1, 0+1), (0+1+1+0+1+1+1, 1+1),$ $(1+0+0+0+1+0+1, 0+1)$
10_{63}	2	$(0+0+0+0+1, 0+1), (0+0+1+0+1, 1+1),$ $(1+0+0+0+1, 0+1)$
10_{65}	3	$(0+0+2+0+0+1+2, 2+1), (0+0+2+0+0+2+2, 1+1),$ $(1+0+0+1+1+1+2, 2+1), (1+0+0+2+1+2+2, 1+1),$ $(1+1+0+2+0+1+2, 2+1), (1+1+1+0+0+1+1, 2+1),$ $(1+2+0+1+0+2+2, 1+1), (1+2+1+0+0+2+1, 1+1),$ $(2+0+2+0+2+0+1, 0+1), (2+1+1+2+1+1+1, 2+1),$ $(2+2+1+1+1+2+1, 1+1)$
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Table 5 Continued from previous page

K	p	$\left(\Delta^{(n)}_{K,\rho}(t), \Delta^{(d)}_{K,\rho}(t)\right)$
10_{67}	5	$(0+0+4+0+4, 0+1), (1+0+0+0+2, 0+1), (2+0+4+1+1, 4+1), (2+0+4+4+1, 1+1), (3+1+0+2+1, 1+1), (3+1+0+2+1, 3+1), (3+4+0+3+1, 2+1), (3+4+0+3+1, 4+1), (4+0+2+0+1, 0+1)$
10_{68}	5	$(0+0+1+0+1, 0+1), (0+0+4+2+1, 2+1), (0+0+4+3+1, 3+1), (1+0+2+0+4, 0+1), (1+0+3+0+1, 0+1), (3+1+2+2+4, 1+1), (3+4+2+3+4, 4+1), (4+2+2+1+1, 1+1), (4+3+2+4+1, 4+1)$
10_{74}	2	$(0+0+0+0+1, 0+1), (0+0+1, 0+1), (0+0+1+0+1, 0+1), (1, 0+1), (1, 1+1)$
10_{75}	2	$(0+0+0+0+1+0+1, 0+1), (0+0+1+0+0+0+1, 0+1), (1+0+1+0+1+0+1, 0+1), (1+0+1+1+0+1+1, 1+1)$
10_{77}	5	$(0+2+3+1+2+1+3, 1+1), (0+2+3+4+2+3+4, 2+1), (0+3+3+1+2+2+4, 3+1), (0+3+3+4+2+4+3, 4+1), (2+0+0+1+0+2+4, 3+1), (2+0+0+4+0+3+4, 2+1), (2+0+4+0+2+1+4, 4+1), (2+0+4+0+2+4+4, 1+1), (2+0+4+0+3+0+4, 0+1), (2+1+4+0+1+4+4, 1+1), (2+4+4+0+1+1+4, 4+1), (3+0+2+0+2+0+4, 0+1), (3+2+2+4+4+4, 2+1), (3+3+2+1+4+1, 3+1)$
10_{87}	5	$(0+0+0+0+1+0+1, 0+1), (0+0+2+0+4+0+4, 0+1), (2+1+3+2+1+0+2, 1+1), (2+2+0+4+3+4+4, 3+1), (2+2+1+0+0+2+4, 4+1), (2+3+0+1+3+1+4, 2+1), (2+3+1+0+0+3+4, 1+1), (2+4+3+3+1+0+2, 4+1), (4+2+0+4+3+4, 2+1), (4+2+1+4+1+0+2, 4+1), (4+3+0+1+3+1, 3+1), (4+3+1+1+1+0+2, 1+1)$
10_{98}	2	$(0+0+0+0+0+0+1, 0+1), (0+0+0+0+1, 0+1), (0+0+1+0+0+0+1, 0+1), (0+0+1+0+1, 1+1), (1+0+0+0+1, 0+1)$
10_{103}	3	$(0+1+0+2+0+2+1, 2+1), (0+2+0+1+0+1+1, 1+1), (1+0+0+0+2+0+1, 0+1), (2+0+1+1+2+2+2, 1+1), (2+0+1+2+2+1+2, 2+1)$
10_{125}	5	$(0+0+3+2+4+0+1, 0+1), (0+0+3+3+4+0+1, 0+1), (0+1+0+2+2+2+1, 1+1), (0+4+0+3+2+3+1, 4+1), (2+1+3+4+4+4+1, 2+1), (2+4+3+1+4+1+1, 3+1), (3+1+0+1+2+1+1, 3+1), (3+4+0+4+2+4+1, 2+1), (4+0+4+0+0+0+1, 0+1), (4+1+2+0+3+4+1, 2+1), (4+4+2+0+3+1+1, 3+1)$
10_{127}	5	$(0+0+4+2+3+3+1, 2+1), (0+0+4+3+3+2+1, 3+1), (0+1+1+4+2+3+1, 0+1), (0+1+4+0+4+3+1, 2+1), (0+4+1+1+2+2+1, 0+1), (0+4+4+0+4+2+1, 3+1), (2+0+2+0+4+0+1, 0+1), (4+2+2+0+0+1+1, 4+1), (4+3+2+0+0+4+1, 1+1)$

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Table 5 Continued from previous page

K	p	$\left(\Delta^{(n)}_{K,\rho}(t), \Delta^{(d)}_{K,\rho}(t)\right)$
10_{129}	5	$(0+0+2+2,2+1), (0+0+2+3,3+1),$ $(1+0+3+2+4,1+1), (1+0+3+3+4,4+1),$ $(1+2+2+4+4,2+1), (1+3+2+1+4,3+1),$ $(2+0+0+0+4,0+1), (3+1+0+2,4+1),$ $(3+4+0+3,1+1)$
10_{130}	5	$(0+1+1+1+4,2+1), (0+1+1+2+4,4+1),$ $(0+4+1+3+4,1+1), (0+4+1+4+4,3+1),$ $(1+0+1+0+4,0+1), (2+2+3+4+3,4+1),$ $(2+3+3+1+3,1+1), (3+2+3,2+1),$ $(3+3+3,3+1)$
10_{131}	5	$(1+0+0+0+4,0+1), (2+2+2,3+1),$ $(2+2+3+3+4,3+1), (2+3+2,2+1),$ $(2+3+3+2+4,2+1), (3+1+2+1+4,1+1),$ $(3+4+2+4+4,4+1)$
	7	$(1+0+6+0+4,0+1), (1+1+0+4+1,2+1),$ $(1+1+1+1+4,4+1), (1+6+0+3+1,5+1),$ $(1+6+1+6+4,3+1), (4+0+0+2+4,1+1),$ $(4+0+0+5+4,6+1), (4+1+5+0+6,5+1),$ $(4+6+5+0+6,2+1), (5+2+3+3+4,5+1),$ $(5+5+3+4+4,2+1)$
10_{132}	5	$(0+1+2+2+1,2+1), (0+1+3+3+1,3+1),$ $(0+4+2+3+1,3+1), (0+4+3+2+1,2+1),$ $(1+0+4+0+1,0+1), (2+2+4+0+1,1+1),$ $(2+3+4+0+1,4+1), (4+1+0+4+1,4+1),$ $(4+4+0+1+1,1+1)$
10_{135}	5	$(0+0+3+0+4,0+1), (3+2+0+3+1,3+1),$ $(3+2+1+3,3+1), (3+3+0+2+1,2+1),$ $(3+3+1+2,2+1), (4+2+2+3+4,4+1),$ $(4+2+4+1+4,3+1), (4+3+2+2+4,1+1),$ $(4+3+4+4+4,2+1)$
10_{136}	2	$(0+0+1+0+1,0+1), (0+0+1+0+1,1+1),$ $(1+0+0+0+1,0+1)$
	3	$(1+0+0+0+1,0+1), (2+0+2+1+1,1+1),$ $(2+0+2+2+1,2+1), (2+1+1+2+1,1+1),$ $(2+2+1+1+1,2+1)$
	5	$(0+0+1+0+1,0+1), (1+0+4+0+1,0+1),$ $(1+2+1+3+1,4+1), (1+3+1+2+1,1+1),$ $(3+2+1+4+1,1+1), (3+3+1+1+1,4+1),$ $(4+1+3+1+1,0+1), (4+2+1+2+1,3+1),$ $(4+3+1+3+1,2+1), (4+4+3+4+1,0+1)$

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Table 5 Continued from previous page

K	p	$\left(\Delta^{(n)}_{K,\rho}(t), \Delta^{(d)}_{K,\rho}(t)\right)$
10_{137}	5	$(0+1+3+3+1, 3+1), (0+4+3+2+1, 2+1),$
		$(1+0+2+1+1, 1+1), (1+0+2+4+1, 4+1),$
		$(1+0+4+0+1, 0+1), (3+2+0+3+1, 0+1),$
		$(3+3+0+2+1, 0+1), (4+1+0+4+1, 4+1),$
		$(4+4+0+1+1, 1+1)$
	7	$(0+0+0+1+1, 2+1), (0+0+0+6+1, 5+1),$
		$(0+2+1+3+1, 4+1), (0+2+4+1+1, 6+1),$
		$(0+5+1+4+1, 3+1), (0+5+4+6+1, 1+1),$
		$(1+2+3+6+1, 1+1), (1+5+3+1+1, 6+1),$
		$(2+0+0+0+1, 0+1), (4+0+2+3+1, 4+1),$
		$(4+0+2+4+1, 3+1), (4+2+1+2+1, 2+1),$
		$(4+5+1+5+1, 5+1), (6+1+0+2+1, 5+1),$
		$(6+1+1+6+1, 6+1), (6+3+2+2+1, 5+1),$
		$(6+4+2+5+1, 2+1), (6+6+0+5+1, 2+1),$
		$(6+6+1+1+1, 1+1)$
10_{140}	2	$(0+0+0+0+1, 0+1), (0+0+1+0+1, 1+1),$
		$(1+0+0+0+1, 0+1)$
	3	$(0+0+0+1+1, 2+1), (0+0+0+2+1, 1+1),$
		$(0+0+1+0+1, 0+1), (1+1+1+1+1, 2+1),$
		$(1+2+1+2+1, 1+1), (2+0+2+1+1, 2+1),$
		$(2+0+2+2+1, 1+1), (2+1+1+2+1, 1+1),$
		$(2+2+1+1+1, 2+1)$
10_{141}	2	$(0+0+1+0+1+0+1, 0+1), (0+1+1+0+1+1+1, 1+1),$
		$(1+0+0+0+1+0+1, 0+1)$
	3	$(0+0+0+0+1+0+1, 0+1), (2+1+0+1+2+0+1, 2+1),$
		$(2+1+2+2+2+0+1, 2+1), (2+2+0+2+2+0+1, 1+1),$
		$(2+2+2+1+2+0+1, 1+1)$
10_{143}	5	$(1+0+4+1+2+4+1, 3+1), (1+0+4+4+2+1+1, 2+1),$
		$(1+1+0+0+1+1+1, 2+1), (1+1+1+2+4+1+1, 2+1),$
		$(1+1+4+1+3+2+1, 4+1), (1+4+0+0+1+4+1, 3+1),$
		$(1+4+1+3+4+4+1, 3+1), (1+4+4+4+3+3+1, 1+1),$
		$(2+0+3+0+3+0+1, 0+1), (2+1+2+2+2+3+1, 1+1),$
		$(2+4+2+3+2+2+1, 4+1), (3+0+1+0+2+0+1, 0+1),$
		$(3+0+4+2+2+3+1, 1+1), (3+0+4+3+2+2+1, 4+1)$
10_{147}	2	$(0+0+1+0+1, 0+1), (0+1+1, 1+1), (1+0+1, 0+1)$
	5	$(0+0+1+0+1, 0+1), (1+0+3+0+4, 0+1),$
		$(2+1+0+2+4, 3+1), (2+4+0+3+4, 2+1),$
		$(3+0+0+0+3, 1+1), (3+0+0+0+3, 4+1),$
		$(3+0+1+1+4, 4+1), (3+0+1+4+4, 1+1),$
		$(3+0+2+0+3, 0+1), (4+1+2+2, 1+1),$
		$(4+1+4+2+2, 4+1), (4+4+2+3, 4+1),$
		$(4+4+4+3+2, 1+1)$

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Table 5 Continued from previous page

K	p	$\left(\Delta^{(n)}_{K,\rho}(t), \Delta^{(d)}_{K,\rho}(t)\right)$
10_{149}	5	$(0+2+4+3+3+0+1, 3+1), (0+3+4+2+3+0+1, 2+1),$ $(2+0+2+2+1+0+1, 1+1), (2+0+2+3+1+0+1, 4+1),$ $(2+0+4+0+2+0+1, 0+1), (2+1+2+1+2+2+1, 0+1),$ $(2+1+2+2+1+0+1, 4+1), (2+2+3+0+4+2+1, 2+1),$ $(2+3+3+0+4+3+1, 3+1), (2+4+2+3+1+0+1, 1+1),$ $(2+4+2+4+2+3+1, 0+1)$
10_{150}	5	$(0+0+4+2+3+3+1, 2+1), (0+0+4+3+3+2+1, 3+1),$ $(0+1+2+3+4+3+1, 2+1), (0+4+2+2+4+2+1, 3+1),$ $(2+0+2+0+4+0+1, 0+1), (4+2+2+0+0+1+1, 4+1),$ $(4+3+2+0+0+4+1, 1+1)$
10_{155}	5	$(0+0+3+0+4+0+1, 0+1), (2+0+2+1+2+2+1, 4+1),$ $(2+0+2+4+2+3+1, 1+1), (2+2+4+4+4+2+1, 4+1),$ $(2+3+4+1+4+3+1, 1+1), (3+1+3+2+1+1+1, 2+1),$ $(3+4+3+3+1+4+1, 3+1), (4+0+0+1+1+2+1, 4+1),$ $(4+0+0+4+1+3+1, 1+1), (4+1+2+4+4+1+1, 2+1),$ $(4+4+2+1+4+4+1, 3+1)$
10_{156}	5	$(0+2+0+0+0+1+1, 4+1), (0+3+0+0+0+4+1, 1+1),$ $(3+0+3+0+0+0+1, 0+1), (3+1+2+2+2+3+1, 2+1),$ $(3+2+0+4+3+4+1, 1+1), (3+3+0+1+3+1+1, 4+1),$ $(3+4+2+3+2+2+1, 3+1), (4+2+1+3+4+3+1, 2+1),$ $(4+3+1+2+4+2+1, 3+1)$
10_{163}	5	$(1+0+3+2+4, 1+1), (1+0+3+3+4, 4+1),$ $(1+2+2+4+4, 2+1), (1+3+2+1+4, 3+1),$ $(2+0+0+0+4, 0+1), (2+1+4+0+1, 2+1),$ $(2+4+4+0+1, 3+1), (3+0+0+2+3, 2+1),$ $(3+0+0+3+3, 3+1), (3+0+1+1+4, 1+1),$ $(3+0+1+4+4, 4+1)$
10_{164}	2	$(0+0+0+0+1+0+1, 0+1), (0+1+1+0+1+1+1, 1+1),$ $(1+0+0+0+1+0+1, 0+1)$
	3	$(0+0+0+0+1+0+1, 0+1), (0+1+0+2+0+1+1, 2+1),$ $(0+1+1+2+2+1+1, 2+1), (0+2+0+1+0+2+1, 1+1),$ $(0+2+1+1+2+2+1, 1+1), (1+1+0+0+1+1+1, 2+1),$ $(1+1+0+1+2+2+1, 1+1), (1+1+1+0+2+2+1, 1+1),$ $(1+2+0+0+1+2+1, 1+1), (1+2+0+2+2+1+1, 2+1),$ $(1+2+1+0+2+1+1, 2+1)$
10_{165}	2	$(0+0+0+0+1, 0+1), (1+0+1+0+1, 0+1),$ $(1+1+0+1+1, 1+1)$
10_{166}	5	$(0+0+4+0+4, 0+1), (1+0+0+0+4, 1+1),$ $(1+0+0+0+4, 4+1), (3+0+0+0+4, 0+1),$ $(3+0+0+0+4, 2+1), (3+0+0+0+4, 3+1),$ $(3+1+3+2+2, 4+1), (3+2+3+3+3, 3+1),$ $(3+3+3+2+3, 2+1), (3+4+3+3+2, 1+1)$

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Department of Mathematical and Computing Sciences, Tokyo Institute of Technology, 2-12-1-W8-43 Oh-okayama, Meguro-ku, Tokyo 152-8552, Japan
E-mail address: kitano@is.titech.ac.jp

Graduate School of Mathematical Sciences, The University of Tokyo, 3-8-1 Komaba, Meguro-ku, Tokyo, 153-8914 Japan
E-mail address: macky@ms.u-tokyo.ac.jp

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ADDRESS:
Graduate School of Mathematical Sciences, The University of Tokyo
3–8–1 Komaba Meguro-ku, Tokyo 153-8914, JAPAN
TEL +81-3-5465-7001 FAX +81-3-5465-7012