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Research field: Algebraic geometry

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Present research:

The notion of derived categories of coherent sheaves on algebraic varieties was originally introduced in order to generalize the duality of sheaf cohomologies (Serre duality), and its origin is pure mathematics. However since Kontsevich proposed Homological mirror symmetry conjecture in 1994, the derived category of coherent sheaves has been recognized as not only a technical tool but also an essential notion in describing the space in string theory. Furthermore, based on ideas from string theory, interesting dualities between several fields of mathematics (representation theory, symplectic geometry, birational geometry, etc.) have been described using the derived category.

I have been interested in giving applications to problems in algebraic geometry via the above dualities. Recently I have established the theory of ‘counting invariants’ of semistable objects in the derived category, and studied how these invariants behave under change of stability conditions. Combined with the duality via derived categories, for instance, it is now possible to compare curve counting invariants of birationally equivalent Calabi-Yau 3-folds. Also I have found an application to the conjecture that the generating series of Donaldson-Thomas invariants is an expansion of a rational function (rationality conjecture by Maulik-Nekrasov-Okounkov-Pandharipande.)

On the other hand, the theories of derived categories and stability conditions on them are not yet fully developed. The future subject is to establish a geometry and an analogy of minimal model theory based on derived categories, and constructing interesting structures on the spaces of stability conditions (automorphic forms, Frobenius structure, etc.)

Notice for the students:

First, please master fundamental knowledge of mathematics which you learn in undergraduate course. It is great if you also master basics of algebraic geometry. The book ‘Algebraic Geometry’ by Hartshorne is a standard textbook. Then it is important not only to learn several fields of mathematics but also to find a research subject which will be your core problem, and consider about this deeply. For this purpose, please be accustomed to thinking math problems by yourself, in your mathematics life.