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Research Fields : Differential Geometry, Complex Geometry/Complex Analysis, Geometric Analysis

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Present Research :

A way to study the intrinsic properties of differentiable manifolds is to use geometric structures to express the differences of the manifolds. Most familiar such structures are the complex structure, the Riemannian structure and the symplectic structure. The Kähler structure carries these three structures at the same time, and they behave consistently with each other. If one wishes to find an optimal structure on a given differentiable manifold the problem is reduced to a differential equation. It is then necessary to study how the existence or non-existence of the solution to the differential equation is reflected by topological or algebraic properties of the manifold. Along such lines of thoughts I have been studying, intensively or less intensively, various topics. But the main theme of my research has been all the way the existence problem of Kähler-Einstein metrics, or more generally the existence problem of Kähler metrics of constant scalar curvature. This problem is at present recognized as a problem to express the necessary and sufficient condition for the existence of a non-linear partial differential equation by using an invariant lying on the intersection of differential geometry, algebraic geometry and topology, and is formulated as a conjecture called Yau-Tian-Donaldson conjecture.

An effective method to find an optimal structure is to run a geometric flow and find a stationary point. Most typical such geometric flows are the Ricci flow and the mean curvature flow. The Ricci flow is well-known for being used in the proof of Poincare conjecture, but is also used successfully for the proof of the sphere theorem and the proof of the existence of Kähler-Einstein metrics of zero and negative Ricci curvature. The analysis of finite time singularity is interesting in itself, and there are active studies on self-similar solutions obtained by rescaling at the singularities.

Notice for the students :

Differential geometry is a branch of mathematics which studies geometric structures on manifolds. The equations that describe the properties of a structure can be regarded as differential equations. Therefore one can say that differential geometry is a branch of mathematics that studies analysis on manifolds. Since geometric structures reflect topological and algebraic properties of manifolds one inevitably has to invoke knowledges from topology, algebraic geometry and real and complex analysis. Then one must need much knowledges from various areas. But no concise package of knowledges for the preparation for future study is not available. I recommend you to find an interesting problem to challenge first, and then to get knowledge to carry it out to solve your problem. Knowledge will help you a lot, but what is required finally is the strength of your will for thorough understanding.