平成２６年度　東京大学大学院

数理科学研究科　数理科学専攻　修士課程

英　語　（筆記試験）

平成２５年　９月２日（月）
１０：４０　～　１２：００

問題は全部で２題ある．２題とも解答すること．

（１）　解答しようとする各問ごとに解答用紙を１枚使用すること．
各解答用紙の所定欄に各自の**氏名**，**受験番号**と解答する**問題の番号**を記入すること．

（２）　草稿用紙の上部に各自の**受験番号**を明記すること．ただし氏名を記入してはならない．

（３）　試験終了後に提出するものは，１題につき１枚，計**２枚の答案**，および**草稿用紙**である．着手した問題数が２題にみたない場合でも，氏名と受験番号のみを記入した白紙の答案を補い，２枚とすること．
指示に反したもの，**答案が２枚でないものは無効**とする．

（４）　解答用紙の裏面を使用する場合は，表面の右下に「裏面使用」と明記すること．

E 第1問

(1) 次の英文を和訳せよ．ただし数学記号はそのまま訳文に挿入すること．

Theorem. For every power series

$$a_0 + a_1 z + a_2 z^2 + \cdots + a_n z^n + \cdots$$

there exists a number R, $0 \leq R \leq \infty$, called the *radius of convergence*, with the following properties:

(i) The series converges absolutely for every z with $|z| < R$. If $0 \leq \rho < R$ the convergence is uniform for $|z| \leq \rho$.

(ii) If $|z| > R$ the terms of the series are unbounded, and the series is consequently divergent.

(iii) In $|z| < R$ the sum of the series is an analytic function. The derivative can be obtained by termwise differentiation, and the derived series has the same radius of convergence.

The circle $|z| = R$ is called the *circle of convergence;* nothing is claimed about the convergence on the circle. We shall show that the assertions in the theorem are true if R is chosen according to the formula

$$1/R = \limsup_{n\to\infty} \sqrt[n]{|a_n|}.$$

This is known as *Hadamard's formula* for the radius of convergence.

[出典] Lars V. Ahlfors, "Complex analysis: An introduction to the theory of analytic functions of one complex variable" third edition. McGraw-Hill, New York (1979), pp 38-39. (一部改変)

(2) 次の英文の下線部を和訳せよ．

We mathematicians need to put far greater effort into communicating mathematical *ideas*. To accomplish this, we need to pay much more attention to communicating not just our definitions, theorems, and proofs, but also our ways of thinking. We need to appreciate the value of different ways of thinking about the same mathematical structure.

We need to focus far more energy on understanding and explaining the basic mental infrastructure of mathematics—with consequently less energy on the most recent results. This entails developing mathematical language that is effective for the radical purpose of conveying ideas to people who don't already know them.

Part of this communication is through proofs.

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When people are doing mathematics, the flow of ideas and the social standard of validity is much more reliable than formal documents. People are usually not very good in checking *formal correctness* of proofs, but they are quite good at detecting potential weaknesses or flaws in proofs.

To avoid misinterpretation, I'd like to emphasize two things I am *not* saying. First, I am *not* advocating any weakening of our community standard of proof; I am trying to describe how the process really works. Careful proofs that will stand up to scrutiny are very important. I think the process of proof on the whole works pretty well in the mathematical community. The kind of change I would advocate is that mathematicians take more care with their proofs, making them really clear and as simple as possible so that if any weakness is present it will be easy to detect. Second, I am *not* criticizing the mathematical study of formal proofs, nor am I criticizing people who put energy into making mathematical arguments more explicit and more formal. These are both useful activities that shed new insights on mathematics.

...

I think that mathematics is one of the most intellectually gratifying of human activities. Because we have a high standard for clear and convincing thinking and because we place a high value on listening to and trying to understand each other, we don't engage in interminable arguments and endless redoing of our mathematics. We are prepared to be convinced by others. Intellectually, mathematics moves very quickly. Entire mathematical landscapes change and change again in amazing ways during a single career.

[注]
gratifying = pleasing and giving satisfaction
interminable = lasting a very long time and therefore boring or annoying

[出典] William P. Thurston "On Proof and Progress in Mathematics" in "18 unconventional essays on the nature of mathematics". Reuben Hersh (ed.), New York, Springer, 2006, pp 45-47. (一部省略) (初出 Bull. Amer. Math. Soc. 30 (1994), 161-177.)

E 第2問

次の和文を英訳せよ．

(1) a, b, c, d を $ad - bc \neq 0$ をみたす複素数とする．このとき，有理関数 $w = \frac{az+b}{cz+d}$ は，リーマン球 $\hat{\mathbb{C}} = \mathbb{C} \cup \{\infty\}$ から $\hat{\mathbb{C}}$ 自身への写像を定める．これを一次分数変換と呼ぶ．

(2) 円 $C : |z - z_0| = R\,(R > 0)$ を考えよう．z_0 と異なる点 z について，z_0 を始点として z を通る半直線上の点であり，$|z^* - z_0||z - z_0| = R^2$ を満たす点を z^* と定める．z^* を C に関する z の共役点という．$z = z_0$ および ∞ の C に関する共役点は，それぞれ ∞ および z_0 であるとする．

(3) 直線 ℓ に関しては，z の ℓ に関して対称な点を共役点 z^* と定める．このとき，明らかに z^* がある円または直線に関して z の共役点であるならば，z は z^* の共役点である．

(4) z^* が直線 ℓ に関して $z(\neq z^*)$ の共役点であるとき，2点 z^*, z を通る直線は ℓ と直交する．円 C に関する共役点に対しては，z^* が円 C に関して $z(\neq z^*)$ の共役点であるとき，2点 z^*, z を通る任意の円は C と直交する．

[注]
リーマン球：Riemann sphere
一次分数変換：linear fractional transformation
共役点：conjugate point