Addendum to “On Isolated Log Canonical Singularities with Index One”

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Abstract. We add a supplementary argument to the paper: O. Fujino, On isolated log canonical singularities with index one.

In this short note, we will freely use the notation in [F]. As Masayuki Kawakita pointed out it, it does not seem to be obvious that the statement in Remark 5.3 in [F] directly follows from the proof of Theorem 5.2 in [F]. It is because $V'_1 \cap V'_2$ in Step 3 in the proof of Theorem 5.2 is not necessarily connected. Therefore, we would like to add the following proposition between Theorem 5.2 and Remark 5.3 in [F]. Note that the proof of Theorem 5.2 and Remark 5.3 in [F] are both correct. We just add a supplementary argument for the reader’s convenience. We note that Remark 5.3 is indispensable for the proof of Theorem 5.5 in [F], where we prove that our invariant $\mu$ coincides with Ishii’s Hodge theoretic invariant.

**Proposition.** If $V'_1 \cap V'_2$ is disconnected, equivalently, has two connected components $W'_1$ and $W'_2$, in Step 3 in the proof of Theorem 5.2, then

$$
\mathbb{C} \cong H^{m-1}(W'_i, \mathcal{O}_{W'_i}) \xrightarrow{\delta|_{W'_i}} H^m(V', \mathcal{O}_V') \cong \mathbb{C}
$$

is an isomorphism for $i = 1, 2$, where $\delta$ is the connecting homomorphism of the Mayer–Vietoris exact sequence.

**Proof.** We note that $H^{m-1}(W'_i, \mathcal{O}_{W'_i}) \cong \mathbb{C}$ for $i = 1, 2$ by Theorem 5.2. We also note that $H^m(V'_i, \mathcal{O}_{V'_i}) = 0$ for $i = 1, 2$ by Step 3 in the proof of Theorem 5.2. We consider the following Mayer–Vietoris exact sequence

$$
\cdots \to H^{m-1}(V'_1, \mathcal{O}_{V'_1}) \oplus H^{m-1}(V'_2, \mathcal{O}_{V'_2}) \xrightarrow{\alpha} H^{m-1}(W'_1, \mathcal{O}_{W'_1}) \oplus H^{m-1}(W'_2, \mathcal{O}_{W'_2}) \xrightarrow{\delta} H^m(V', \mathcal{O}_V') \to 0
$$

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as in Step 3 in the proof of Theorem 5.2. Note that \( \text{Im} \alpha \simeq \text{Ker} \delta \) is a one-dimensional \( \mathbb{C} \)-vector space. We consider the exact sequence:

\[
\cdots \to H^{m-1}(V'_1, \mathcal{O}_{V'_1}) \to H^{m-1}(W'_i, \mathcal{O}_{W'_i}) \to H^m(V'_1, \mathcal{O}_{V'_1}(-W'_i)) \to 0.
\]

By the Serre duality, \( H^m(V'_1, \mathcal{O}_{V'_1}(-W'_i)) \) is isomorphic to \( H^0(V'_1, \mathcal{O}_{V'_1}(K_{V'_1} + W'_i)) \) for \( i = 1, 2 \). We can check that \( H^0(V'_1, \mathcal{O}_{V'_1}(K_{V'_1} + W'_i)) = 0 \) for \( i = 1, 2 \) by the same way as in Step 3 in the proof of Theorem 5.2. Therefore, the natural map, which is induced by the restriction,

\[
H^{m-1}(V'_1, \mathcal{O}_{V'_1}) \to H^{m-1}(W'_i, \mathcal{O}_{W'_i}) \simeq \mathbb{C}
\]

is surjective for \( i = 1, 2 \). Thus, we see that

\[
\text{Im} \alpha \simeq \mathbb{C} \left( \subset H^{m-1}(W'_1, \mathcal{O}_{W'_1}) \oplus H^{m-1}(W'_2, \mathcal{O}_{W'_2}) \simeq \mathbb{C}^2 \right)
\]

contains neither \( H^{m-1}(W'_1, \mathcal{O}_{W'_1}) \simeq \mathbb{C} \) nor \( H^{m-1}(W'_2, \mathcal{O}_{W'_2}) \simeq \mathbb{C} \). This implies that

\[
\mathbb{C} \simeq H^{m-1}(W'_1, \mathcal{O}_{W'_1}) \xrightarrow{\delta|_{W'_1}} H^m(V'_1, \mathcal{O}_{V'_1}) \simeq \mathbb{C}
\]

is non-trivial, equivalently, an isomorphism, for \( i = 1, 2 \). \( \Box \)

The statement in [F, Remark 5.3] follows from Step 3 in the proof of [F, Theorem 5.2] and Proposition.

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\textbf{References}

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