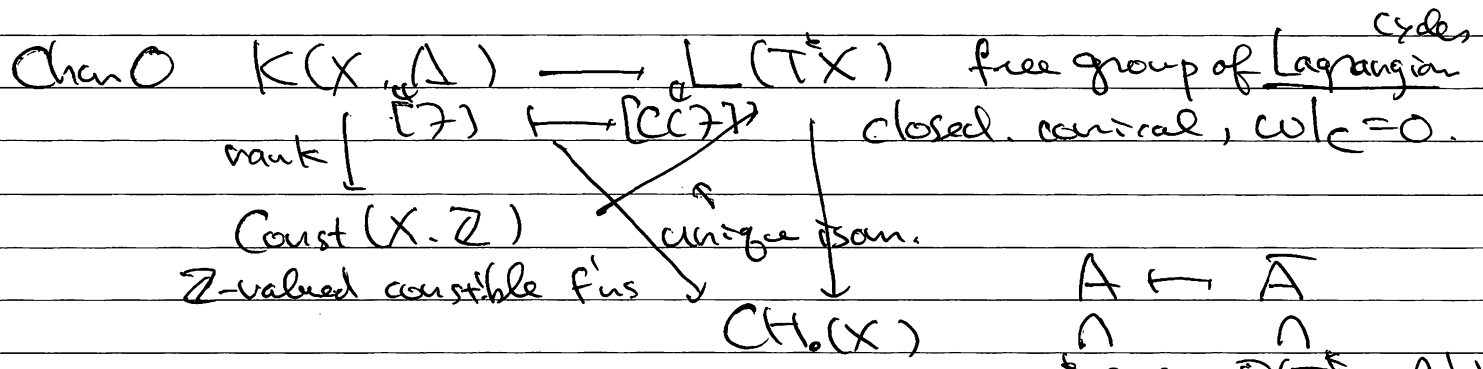


Characteristic cycle and restriction to curves

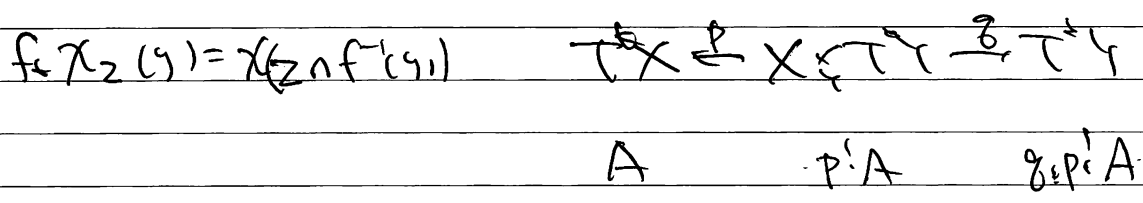
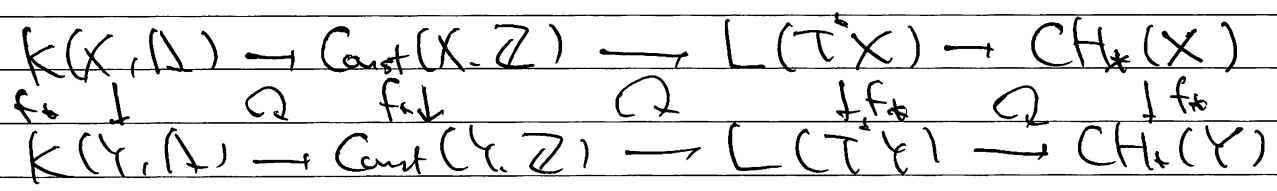
k_2 perfect X smooth (k_2)
 Λ finite field $l = \dim \Lambda \neq p = \dim k_2 \geq 0$

$K(X, \Lambda)$ Grothardieck group of constructible sheaves of Λ -mod. on X .



composition MacPherson's Chern class

proper push forward. $f: X \rightarrow Y$ proper morphism



Chang $K(X, \Lambda) \rightarrow ZC_n(T^*X)$ closed conical dim $n = \dim X$

No CD $K(X, \Lambda) \rightarrow ZC_n(T^*X)$
 \downarrow
 $\text{Const}(X, \mathbb{Z})$

$\dim X = 1$
 $CC \tau = -(\text{rk } \tau)(\omega - [T^*X]) + \sum_{x \in \mathbb{O}} a_x(\tau) [T_x^*X]$

Austin conductor
 $a_x \tau = \text{rk } \tau|_{\eta_x} - \text{rk } \tau|_x + \text{Sw}_x \tau \leftarrow \text{wild ramification}$

$C_1(X)$ suggested by Beilinson
 group of \mathbb{Z} -valued fns on the set of $(v \in \bar{C} \rightarrow C \hookrightarrow X)$

$C \hookrightarrow X$ closed integral curve
 $\bar{C} \rightarrow C$ normalization v - closed pt

$$\begin{array}{ccc}
 K(X, \Lambda) & \rightarrow & \text{Const}(X) \oplus C_1(X) \\
 \uparrow & \longleftarrow & (\text{rank } \mathbb{Z}, (v, c) \mapsto \text{Sw}_x \mathbb{Z}(\bar{c})) \\
 \downarrow & & \\
 C(X) & & \text{image}
 \end{array}$$

Theorem 1. There exists a unique morphism $\hat{C}(X) \rightarrow ZC_n(T^*X)$
 s.t. the diagram

$$\begin{array}{ccc}
 K(X, \Lambda) & \xrightarrow{\text{cc.}} & ZC_n(T^*X) \\
 \downarrow & & \uparrow \\
 \hat{C}(X) & &
 \end{array}$$

is commutative.

Variant for $\Lambda \neq \Lambda', (l \neq l')$

Preceding results

S. Yagawa, H. Kato.

Larger gts of $K(X, \Lambda)$

Proper pushforward $f: X \rightarrow Y$ proper $n = \dim X, m = \dim Y$

$$\begin{array}{ccccccc}
 K(X, \Lambda) & \rightarrow & \hat{C}(X) & \rightarrow & ZC_n(T^*X) & \rightarrow & CH_n(X) \\
 f_* \downarrow & & \downarrow & & \downarrow & & \\
 K(Y, \Lambda) & \rightarrow & \hat{C}(Y) & \rightarrow & ZC_m(T^*Y) & \rightarrow & CH_m(Y)
 \end{array}$$

Theorem 2. There exists a unique morphism $f_*: \hat{C}(X) \rightarrow \hat{C}(Y)$
 s.t. the diagram is commutative

Preceding results.

Vidal, Yagawa, H. Kato.

Larger gts of $K(X, \Lambda)$

$$\begin{array}{ccc}
 \text{No CD} & K(X, \Lambda) & \rightarrow ZC_n(T^*X) \\
 & f_* \downarrow & \downarrow \\
 & K(Y, \Lambda) & \rightarrow ZC_m(T^*Y)
 \end{array}$$

Example $X = Y = A^2$ $\varphi = \sum_{p=0}^{\infty} b_p [x, y] > D$ $x=0$
 $v = \varphi - D.$

$f: X \rightarrow Y$ Antin-Schneier covering $t^p - t = \frac{y}{x^{ph}}$ $w = x^n t$

$w^p - x^{(p-1)n} w = y$ $X = \sum_{p=0}^{\infty} b_p [x, w] > D'$ $x=0$

h, m integers satisfying $ph > m > ph - (p-1)n, p \geq 1$ ($h > n$)
 $pt + m$ ($> h$)

χ $t^p - t = \frac{y}{x^{ph}}$, φ $t^p - t = \frac{1}{x^m}$ char on V .

$$CC \chi = CC(X \circ \varphi) = [T_{\varphi}^* \chi] + ph \langle \text{cls} \rangle_D.$$

$$\neq CC \varphi = [T_{\varphi}^* \chi] + m [T_D^* \chi] \langle \text{cls} \rangle_D$$

$$f^* \chi \quad t^p - t = \frac{w^p - x^{(p-1)n} w}{x^{ph}} \sim \frac{w}{x^h} - \frac{w}{x^{ph - (p-1)n}} \quad m > h$$

$$CC(f^* \chi \circ \varphi) = CC(f^* \varphi) = [T_{\chi}^* \chi] + m [T_D^* \chi]$$

$$CC(f_+ f^* \chi \circ \varphi) = p CC(X \circ \varphi) \neq p CC \varphi = CC(f_+ f^* \varphi).$$

No CD $K(X, \Lambda) \rightarrow CH(X)$
 $\downarrow f_*$ \downarrow
 $K(Y, \Lambda) \rightarrow CH(Y)$

Counterexample Frobenius.

However

Conjecture $K(X, \Lambda) \rightarrow CH_0(X)$
 $\downarrow f_*$ \cong $\downarrow f_*$
 $K(Y, \Lambda) \rightarrow CH_0(Y)$

If X, Y proj / \mathbb{Z} finite Umemaki-Yang-Zhao.

Outline of Proof of Theorems 1 & 2.

- Crucial case $\dim X = 2, \dim Y = 1$
- Induction on \dim .

Thm 2 By proper b.c. $\dim Y = 1$.
 devissage red. to $\dim X = 2, \dim Y = 1$

Thm 1. CC is characterized by the Milnor formula for the total dimension of vanishing cycles.
 To replace p.b.c. use nearby cycles over gen/basis.

$$\begin{array}{ccc} Z \subset X & \xrightarrow{g} & \mathbb{A}_Y^1 \\ & \searrow f & \downarrow p \\ & & Y \end{array} \quad R\Phi_f = R\rho_* \circ R\Phi_g$$

g is transversal (\Rightarrow locally acyclic)
 away from Z quasi-finite over Y
 \Downarrow
 $R\Phi_g$ coincide with b.c.

X normal $U \subset X$ dense open $j: U \hookrightarrow X$ open immersion
 2. f' loc.-const const'ble on U

f & f' have the same wild ram'ns (cf. Deligne-Illusie, Gabber & Ullmann)

if \exists propn $X' \rightarrow X$ ism on U . X' normal. s.t
 $\forall x' \in X' - U \quad \forall \sigma \in I_{x'}$ inertia of order power of p
 we have

$$\dim f_{\eta_x}^{\langle \sigma \rangle} = \dim f'_{\eta_x}^{\langle \sigma \rangle}$$

f & f' have the same rk at a closed pt x .

if $rk(f) = rk(f')$ & \forall closed integral curve $C \hookrightarrow X \quad \forall v \in \bar{C} \rightarrow C$
 normalization above x , we have

$$Sw_n f|_{\bar{C}} = Sw_n f'|_{\bar{C}}$$

Key Proposition Assume $\dim X = 2$

Same rk + cond at $x \Rightarrow$ Same wild ram'ns on a nbd of x

Key Prop + Preceding results \Rightarrow $\dim X = 2, \dim T \geq 1$ case of Thm 1 & 2.

Proof of Key Prop.

- First prove the case $rk(f) = rk(f') = 1$, char of order p
 Kato's cleaning theorem

- This case implies

• univ. Same rk + cond at $x \Rightarrow$ Same wild ram on a nbd of x
 replace $C \hookrightarrow X$ by $C' \hookrightarrow X$
 not necessarily ism

Same rk + cond at x
 deformation of curve. Stability of the Swan conductor
 use FLK.