斎 藤 毅 (SAITO Takeshi)

A. 研究概要

前年度, 加藤 和也氏との共同研究により, 高次元における Bloch の導手公式を, かなり一般の場合に証明することができた. 今年はその証明を論文にまとめた. なかなか集中して時間をとることができず, 論文を完成させられなかったのが残念である. しかし, 論文を書いている途中で, 次の 2 点について進歩があった. 1 つは導手公式の代数対応への一般化である. 本来の導手公式はこの観点からは, 代数対応が対角であるという特別な場合になる. 一般に局所体 K 上の固有非特異代数多様体 X_K からそれ自身への代数多様体 Γ に対し, その Swan 導手 $\mathrm{Sw}(X_K, \Gamma)$ が, ℓ 進コホモロジーにひきおこされる自己準同型 Γ^* を使って定義される. これが ℓ によらない整数であり, しかも局所化された対数交点数 $[[\Gamma, X]]$ と等しいことを示すことができた. この一般化された公式は, 多様体の自己準同型にたいして適用できるので, 将来導手公式を係数層つきの場合へと拡張するためにも重要な結果である. 証明は本来の導手公式の証明の自然な拡張である.
もう 1 つは, Bloch-Abbes により定義されていた Chow 群を使った局所化された交点理論と, K 理論的な局所化された交点理論の関係を証明したことである. これは過度交点積公式がどちらの理論でも同じ形をしていることから従う.
その他, 剰余体が一般の離散付値体の分岐理論についても A.Abbes 氏と共同で研究したが, 部分的な結果がえられただけで, まとまった成果はえられなかった.

Last year, I proved the conductor formula of Bloch in higher dimension under a mild hypothesis in a joint work with K.Kato. This year, I wrote an article on this proof. Unfortunately, it is not yet completed. While writing it, I made progress on the following two points. One is a generalization of the conductor formula to algebraic correspondences. The original conductor formula is the special case where the correspondence is the diagonal. For an algebraic correspondence Γ on an proper smooth scheme X_K over a local field K, it Swan conductor $\mathrm{Sw}(X_K, \Gamma)$ is defined by using the induced endomorphism Γ^* on ℓ-adic etale cohomology. I have shown that it is an integer independent of ℓ and is equal to the logarithmic localized intersection number $[[\Gamma, X]]$. This generalized formula is applicable to an endomorphism of a variety and should be useful in a potential generalization of the conductor formula with coefficient sheaves. The proof is parallel to that of the original conductor formula.
The other is a relation between the localized intersection theory using Chow groups defined by Bloch-Abbes our localized intersection theory using K-groups. It follows form that the excess intersection formula has the same form in the both theory.
I also studied ramification of complete discrete valuation fields with imperfect residue field. I only succeeded to obtain some partial results.

B. 発表論文

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C. 口頭発表

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D. 講義

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2. 数理科学 I: 2 変数関数の微積分続論 (教養学部前期課程講義)
3. Bloch の導手公式: 局所体上の代数多様体のエタール・コホモロジーについて解説した (集中講義 九州大学 2000 年 5 月)

E. 修士・博士論文

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F. 対外研究サービス

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H. JSPS ビジター

Thomas Geisser

During the year 2000 I was working on two joint projects with Lars Hesselholt (MIT, USA), who came to visit me in Tokyo to give talks and to work with me.

The first project is a calculation of the K-theory with p-adic coefficients of power series rings $R = k[[T_1, ..., T_n]]$ over fields k of characteristic p. Previously, only trivial cases had been known (like K_i for $i \leq 3$). We were able to show that if k has a finite p-base, then the p-adic K-theory spectrum $K(R, Z_p)$ of R agrees with the homotopy limit of the K-theory spectra $K(R/I^j, Z_p)$ of R/I^j, where I is the maximal ideal of R. In partiular, the K-groups of R can be expressed in terms of the K-groups of the R/I^j. In the proof one uses on the hand that in the situation above, K-theory and Milnor K-theory agree (this has been proven by Marc Levine and myself), and on the other hand the connection between K-theory, topological cyclic homology and the de Rham Witt complex (which has been established by Hesselholt). A preprint on this work is available, and has been submitted for publication.

In the second project, Hesselholt and I compare K-theory and topological cyclic homology on the closed fiber of a smooth scheme over a discrete valuation ring] V of mixed characteristic $(0, p)$. Our main theorem is that for a Henselian local ring A of such a scheme at a point of the closed fiber, K-theory and topological cyclic homology agree above the dimension of A, and they always agree if A is stricly Henselian. In partiular, etale K-theory and topological cyclic homology agree for a smooth and proper scheme over V. The theorem is known for A/p instead of A by previous joint work of Hesselholt and myself. We generalize the "calculus of functors", developed by Goodwillie, to show that this implies the theorem for A/p^j for any j. Finally, we generalize a theorem of Suslin to show that both theories commute with inverse limits, to conclude the theorem for A. Again, a preprint is available, and has been submitted for publication.