10/25
例:　単位球の体積

$$2\int_{x^2+y^2\leq 1}\sqrt{1-x^2-y^2}dxdy = \int_{-1}^{1}dx\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}2\sqrt{1-x^2-y^2}dy$$
$$= \int_{-1}^{1}\pi(1-x^2)dx = \pi[x-\frac{x^3}{3}]_{-1}^{1} = \frac{4\pi}{3}.$$

$$\int_{0\leq y\leq x\leq\pi} y\cos(x-y)dxdy = \int_0^{\pi}dy\int_y^{\pi}y\cos(x-y)dx = \int_0^{\pi}y[\sin(x-y)]_y^{\pi}dy$$
$$= \int_0^{\pi}y\sin ydy = [-y\cos y]_0^{\pi} - \int_0^{\pi}-\cos ydy = \pi.$$
$$= \int_0^{\pi}dx\int_0^{x}y\cos(y-x)dy = \int_0^{\pi}([y\sin(y-x)]_0^{x} - \int_0^{x}\sin(y-x)dy)dx$$
$$= \int_0^{\pi}(\int_0^{x}\sin ydy)dx = \int_0^{\pi}[-\cos y]_0^{x}dx = \int_0^{\pi}1-\cos xdx = \pi.$$

計算法 II.　変数変換.
1 変数の場合。
平面の座標変換。極座標。

$$\int_D f(x,y)dxdy = \int_E f(r\cos\theta, r\sin\theta)rdrd\theta.$$

単位球の体積

$$2\int_{x^2+y^2\leq 1}\sqrt{1-x^2-y^2}dxdy = 2\int_{0\leq r\leq 1, 0\leq\theta\leq 2\pi}\sqrt{1-r^2}rdrd\theta$$
$$= 2\int_0^1 dr\int_0^{2\pi}\sqrt{1-r^2}rd\theta = 4\pi\int_0^1\sqrt{1-r^2}rdr = 4\pi[-\frac{1}{3}\sqrt{1-r^2}^3]_0^1 = \frac{4\pi}{3}.$$