河 東 泰 之 (KAWAHIGASHI Yasuyuki)

A. 研究概要

今年度はまず，佐藤信哉，和久井道久との共同研究で，unitary $6j$-symbol から生じる 3 次元 topological quantum field theory について調べた．Turaev-Viro-Ocneanu の構成法によって，unitary $6j$-symbol から 3 次元閉多様体の複素数値位相不変量を作ることができる．一方，unitary $6j$-symbol に対して，Drinfel′d の quantum double 構成法の一般化が Ocneanu によって考えられているので，これを適用すれば modular tensor category が作り出せる．すると，そこから，Reshetikhin-Turaev の構成法でやはり，3 次元閉多様体の複素数値位相不変量を作ることができる．当然，この二つの位相不変量の関係が問題になるが，両者が一致することを我々は証明した．Unitary $6j$-symbol は作用素環論における subfactor から生じることが Ocneanu によって知られており，通常の量子群論で知られているもののほかにもいくつか例が構成されている．佐藤，和久井はこの結果を用いて多くの例について具体的な計算を行っている．

ついで，Longo と，作用素環の 2 次元共形ネットの分類について研究した．「運動群」が微分同相写像群である場合は，実数に値を持つ不変量 “central charge” が定義できることに基づき，central charge が 1 未満の場合の完全分類を与えた．1 次元共形ネットの場合について，同様の分類理論を Longo と昨年度に得ており，そこでも，A-D-E 型 Dynkin 図形の対が現れるが，1 次元の場合は，D_{2n+1}, E_7 が現れなかったのに対し，2 次元ではこれらも分類表に現れる．1 次元の場合の分類結果を用いるのだが，tensor category の 2-cohomology 群が新たな研究対象として自然に現れる点が新しい．Virasoro algebra に関連した tensor category について，この cohomology 群の消滅定理を示すことが重要なステップとなる．

We first studied 3-dimensional topological quantum field theories arising from unitary $6j$-symbols in a joint work with N. Sato and M. Wakui. The Turaev-Viro-Ocneanu construction gives a numerical invariant of closed 3-manifolds from such $6j$-symbols. Ocneanu has a quantum double construction for such $6j$-symbols as an analogue of the quantum double construction of Drinfel′d, and it produces a modular tensor category. Then by applying the Reshetikhin-Turaev construction to this tensor category, we also obtain a numerical invariant of closed 3-manifolds. It is a natural problem to study a relation between these two invariants, and we have proved that they are identical. Ocneanu showed that such unitary $6j$-symbols arise from subfactors and there have been constructed such $6j$-symbols, not arising directly from quantum groups. Sato and Wakui have used this result for explicit computations of this invariant.

Next, we obtained a classification result of 2-dimensional local conformal nets of factors with R. Longo. If we have diffeomorphism covariance, we can define a central charge, as a numerical invariant of such nets, and in the case this number is less than 1, we have obtained a complete classification of such nets. We had a similar classification result for 1-dimensional nets in the previous year, also with Longo, and we again have pairs of A-D-E Dynkin diagrams as invariants, but now D_{2n+1} and E_7 do appear in the classification, though they did not appear in the classification of 1-dimensional nets. The new feature of the 2-dimensional classification theory is emergence of 2-cohomology groups for tensor categories. Our main technical tool is vanishing of such 2-cohomology groups for certain tensor categories related to the Virasoro algebra.

B. 発表論文

1. J. Böckenhauer, D. E. Evans and Y. Kawahigashi: “Chiral structure of modular invariants for subfactors”, Commun. Math. Phys. **210** (2000) 733–784.

2. Y. Kawahigashi, R. Longo and M. Müger: “Multi-interval subfactors and modularity of representations in conformal field theory”, Commun. Math. Phys. **219** (2001) 631–669.

3. J. Böckenhauer, D. E. Evans and Y. Kawahigashi: “Longo-Rehren subfactors arising from α-induction”, Publ. RIMS, Kyoto Univ. **37** (2001) 1–35.

4. Y. Kawahigashi: "Braiding and extensions of endomorphisms of subfactors", in "Mathematical Physics in Mathematics and Physics", R. Longo ed., The Fields Institute Communications **30**, AMS Publications (2001) 261–269.

5. Y. Kawahigashi: "Generalized Longo-Rehren subfactors and α-induction", Commun. Math. Phys. **226** (2002) 269–287.

6. Y. Kawahigashi and R. Longo: "Classification of local conformal nets: Case $c < 1$", preprint, math-ph/0201015.

7. Y. Kawahigashi: "Conformal quantum field theory and subfactors", to appear in Acta Math. Sin. (special issue).

8. Y. Kawahigashi, N. Sato and M. Wakui: "$(2+1)$-dimensional topological quantum field theory from subfactors and Dehn surgery formula for 3-manifold invariants", preprint, math.OA/0208238.

9. Y. Kawahigashi: "Classification of operator algebraic conformal field theories", preprint, math.OA/0211141.

10. Y. Kawahigashi and R. Longo: "Classification of two-dimensional local conformal nets with $c < 1$ and 2-cohomology vanishing for tensor categories, in preparation.

C. 口頭発表

1. Classification of local conformal nets: case $c < 1$, Operator Algebra Seminar, II Università di Roma (Italy), March 2002.

2. Subfactor 理論とその応用 — 作用素環と場の量子論 —, 日本数学会年会総合講演, 明治大学, 2002 年 3 月.

3. Classification of local conformal nets: case $c < 1$, AMS Summer Research Conference "Advances in Quantum Dynamics", Mount Holyoke College (U.S.A.), June 2002.

4. Classification of local conformal nets: case $c < 1$, Miniworkshop "Index theorems and modularity in operator algebras", Oberwolfach (Germany), June 2002.

5. Classification of local conformal nets: case $c < 1$, "The 19th International Conference on Operator Theory", Timişoara (Romania), July 2002.

6. Classification of local conformal nets: case $c < 1$, "Frontier of Non Commutative Analysis and Mathematical Quantum Theory", Fukuoka (Japan), August 2002.

7. Classification of local conformal nets: case $c < 1$, "Operator Algebras and Applications", Chengde (China), August 2002.

8. Topological quantum field theories arising from operator algebras, "International Workshop on Quantum Field Theory and Noncommutative Geometry", Sendai (Japan), November 2002.

9. Topological invariants of 3-manifolds arising from subfactors, Operator Algebra Seminar, II Università di Roma (Italy), December 2002.

10. Classification of operator algebraic conformal field theories by representation categories, Workshop "Categorical Algebra, Deformation Theory and Field Theory II", Kyoto (Japan), December 2002.

D. 講義

1. 全学自由研究ゼミナール：超準解析の講義. Robinson による超準解析の理論を，基礎から解説した．超フィルターの構成，超冪モデルの構成と種々の性質などを論じた．(教養学部前期課程講義)

2. 数理科学 II：常微分方程式の講義. 常微分方程式の解の存在と一意性，基本的な解法について解説した．(教養学部前期課程講義)

3. 解析学 XF・無限次元構造論：作用素環論の講義. 代数的場の量子論における，Doplicher-Haag-Roberts の理論を基礎から解説した．(数理大学院・4 年生共通講義)

E. 修士・博士論文

1. (課程博士) 勝良 健史 (KATSURA Takeshi): A class of C^*-algebras generalizing both graph algebras and homeomorphism C^*-algebras.

2. (修士) 戸松　玲治 (TOMATSU　Reiji): Amenable discrete quantum groups.

F. 対外研究サービス

1. “Summer School 数理物理 2002: 非可換幾何学と数理物理” を東京大学大学院数理科学研究科において主催, (小嶋泉氏と共同, 2002 年 9 月 21 日〜23 日).

G. 受賞

第 1 回作用素環賞受賞 (2000).
日本数学会賞春季賞 (2002).