ON ANALYTIC CONSTRUCTION OF THE GROUP THREE-COCYCLES

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One of the most important group cocycles is the two-cocycle on the restricted general linear group of a polarised Hilbert space (H, H_+). It has a wide range of applications, like the central extensions of the loop group, theory of Toeplitz operators, gauge theory or invariants of the K_2^{alg}. This two-cocycle can be seen as a two-cocycle associated to the action of the group $GL_{res}(H, H_+)$ on the category of subspaces $K \subset H$ such that the product of orthogonal projections

$$P_{H_+} P_K : K \to H_+$$

is in $\mathcal{L}^2(H)$. Morphisms in this category are given by lines $Det(P_{K_1} P_{K_2})$.

Similarly, given an action of a group G on an n-category satisfying certain conditions, one can construct a (n+1)-cocycle on G. A well known example is the n-Tate space, essentially an algebra of the form $K = k((s_1))((s_2)) \ldots ((s_n))$, where the group is the group of invertibles in K and the n-category structure comes from the natural filtration of K.

The corresponding cocycles, when evaluated on $K_{n+1}^{alg}(K)$, reproduce the Tate tame symbol. However, the constructions are purely algebraic and do not seem to extend to the analytic context, as in the case of $n = 1$.

In this talk we will sketch a construction of a (family of) two-category associated to a pair of commuting idempotents P and Q on a Hilbert space and construct the associated three cocycle on the associated groups. For example, in the case of a two-Tate space, this produces an extension of the Tate symbol from $\mathbb{C}((z_1))((z_2))$ to, say, $C^\infty(\mathbb{T}^2)$, but also a corresponding invariant of K_3^{alg} of the non-commutative torus $C^\infty(\mathbb{T}_\theta^2)$.

The construction is based on the properties of the determinant of Fredholm operators, in particular on the existence of the canonical perturbation isomorphism $Det(T) \simeq Det(S)$ whenever T and S are two Fredholm operators satisfying $T - S \in \mathcal{L}^1(H)$.

This is a joint work with Jens Kaad and Jesse Wolfson.