Restriction of Unitary Representations
— discrete and continuous spectrum

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Let π be an irreducible unitary representation of a group G. A **branching law** is the irreducible decomposition of π with regard to its subgroup G':

$$\pi|_{G'} \simeq \int_{\widehat{G'}}^{\oplus} m_\pi(\tau)\tau \, d\mu(\tau) \qquad \text{(a direct integral)}.$$

Such a decomposition is unique, for example, if G' is a real reductive group.

Special cases of branching problems include and/or reduce to the followings: Littlewood-Richardson rules, the decomposition of tensor product representations, character formulas, Blattner formulas, Plancherel theorems for homogeneous spaces, description of breaking symmetries in quantum mechanics, the theta-lifting in the theory of automorphic forms, etc.

Our concern is with non-compact subgroups G', and we shall explain the algebraic and analytic theory of branching laws without continuous spectrum. Then, we shall discuss its recent applications which include:

1. (Representation theory) Understanding of "singular" representations.

2. (Discontinuous groups) The topology of modular varieties.

3. (L^p-analysis) Construction of new discrete series for homogeneous spaces.