教授 (Professor)

小林 俊行 (KOBAYASHI Toshiyuki)

A. 研究概要

2017–2021 の 5 年間においては，主に以下の 3 テーマの理論構築を行い，総計で約 1,200 ページの論文を著した。

1. **対称性破れ作用素の構成と分類問題**

簡約リー群の無限次元表現の「分岐則」に関して，定性的理論から定量的理論に深化させるプログラムを提起した [11].

1.A.(定性的理論) 離散的分岐則の理論の要となる K'-admissibility の十分条件 ([Ann. Math., 1998] の主定理の 1 つ) が，実は必要十分条件であることをシンプレクティック幾何の手法を用いて証明した [2].

1.B.(定性的理論 2) '小さい' 無限次元表現を部分群に制限したときの重複度が有限・有界になるための判定条件を与え [3,10,13]，特に，対称対に周期をもつ表現の別の対称対への表現が有界重複度をもつケースを**可視的作用**の理論を援用して分類した.

1.C.(定量的理論 1—対称性破れの微分作用素) 対称性破れ作用素の新しい構成法 (F-method) を提唱し，新しい微分作用素の族 (Rankin–Cohen 作用素や Juhl の共形不変な作用素の一般化) を構成した ([Adv. Math. 2015], [Selecta Math. 2016], [Lect. Notes Math. 2016]).

1.D. (定量的理論 2—対称性の破れの分類理論) 対称性破れ作用素を，非局所作用素まで含めて構成し，完全に分類するプログラムを提起し，B. Speh と共同でその最初の成功例を与えた (Memoirs of AMS, 2015, 行列値への拡張は著書 [2], 応用は [9])。また，逆変換として "ホログラフィック変換" の概念を導入した [1], [Ann. Inst. Fourier, 2020].

2. **不連続群**

筆者の長年のモチーフである「リーマン幾何学の枠組を越えた不連続群」について，新しい幾何に対するスペクトル理論の構築に初めて踏み込んだ。幾何学的な準備として，離散群の作用の不連続性を量的に評価する sharpness という概念を導入し，高次元タイヒミュラー空間上で**安定な**離散スペクトラムを構成し，長編の論文 [Adv. Math. 2016] を出版した。さらに，長編の

第 2 論文 [JLT2019] および [Progr. Math. 2017] で隠れた対称性を用いた微分作用素環の構造定理を証明し，それを標準的な擬リーマン局所対称空間のスペクトル解析に活用した [12]。

3. **非対称空間の大域解析**

力学系のアイディアを用いて，等質空間 G/H の正則表現が L^p 緩増加となるための必要十分条件を H が簡約の場合に証明し [J. Euro. Math. 2015]，それを H が一般の場合に拡張した (Y. Benoist と共同, 文献 [4])。さらに 第 3 論文 [4] では非緩増加な簡約型等質空間の完全な分類を行い，第 4 論文 [5] では，緩増加性という解析的な性質が，リー代数の極限に関する位相的性質，シンプレクティック多様体の幾何的量子化という微分幾何的性質，および，凸多面体の組合せ論的性質とそれぞれ同値であることを証明し，さらに [7,8] を著した。

For the last five years, I have been working on the following research topics.

1. **Restriction of representations: Branching problems and symmetry breaking operators**

Branching problems in a broad sense try to understand the behavior of the restriction of irreducible representations to subgroups. I proposed in [Progr. Math., 2015] a general program to advance branching problems for reductive groups, see [11] also for a survey.

1.A Concerning the discretely decomposability of the restriction of representations, I proved in [2] by using symplectic geometry, the converse of one of the main theorems in my previous paper [Ann. Math., 1998].

1.B I proved a criterion for finite multiplicity/bounded multiplicity of the restriction of 'small' infinite-dimensional representations to reductive subgroups in [3,10,13]. In particular, I classified the triples (G, H, G') such that (G, H) is a symmetric pair and that any irreducible H-distinguished representations have bounded multiplicity when restricted to another symmetric pair (G, G').

1.C In the BGG category $\mathcal{O}$, I proposed a new

effective method to find singular vectors ('*F*-method'), and joint with B. Ørsted, V. Souček, P. Somberg, M. Pevzner, and T. Kubo determined explicit formulæ of covariant differential operators in various geometric settings ([Adv. Math. 2015], [Selecta Math. 2016], and [Lect. Notes Math. 2016]).

1.D With B. Speh, I classified *symmetry breaking operators* (SBOs) of principal series for a pair of Lorentz groups (Memoirs of AMS 2015 and book [2]), which give the first successful for the complete classification of SBOs. A part of this work is extended to higher rank case.

1.E As an "inversion" of the symmetry breaking, I introduced the concept of **holographic transform** joint with Pevzner ([Ann. Inst. Fourier 2020], [1]).

2. Analysis on locally symmetric spaces—beyond the Riemannian case

Developing my long motif on discontinuous groups beyond the Riemannian case, I initiated the study on global analysis on locally non-Riemannian symmetric spaces with F. Kassel in [Adv. Math. 2016] and proved the existence of "stable spectrum" under small deformation of discontinuous groups. Further new ideas are proposed in [Progr. Math. 2017], [JLT2019], and [11,12].

3. Analysis on manifolds with group actions

This is a challenge to the global analysis on homogeneous spaces beyond symmetric spaces. Jointly with Y. Benoist [J. Euro. Math. '15], we proved a criterion for L^p-temperedness of the regular representation on G/H in the generality that $G \supset H$ are pair of reductive groups, and in [4] for general H. A complete description of nontempered homogeneous spaces G/H with $H \subset G$ reductive has been accomplished in [5], and a further connection with other disciplines of mathematics has been explored in [6]. Further references include [7,8].

B. 発表論文

(論文は 2021 年以降のもののみを記載する. 2017 年 ～2020 年の論文は，過去の Annual Report の各年度に記載.)

1. T. Kobayashi. Branching laws of unitary representations associated to minimal elliptic orbits for indefinite orthogonal group $O(p,q)$. Advances in Mathematics **388**, (2021), Paper No. 107862. 38 pages. DOI: 10.1016/j.aim.2021.107862.

2. T. Kobayashi. Admissible restrictions of irreducible representations of reductive Lie groups: Symplectic geometry and discrete decomposability. Pure and Applied Mathematics Quarterly **17**, (2021), pp. 1321–1343. Special volume in memory of Bertram Kostant.

3. T. Kobayashi. Bounded Multiplicity Theorems for Induction and Restriction. Journal of Lie Theory, **32**, (2022), pp. 197–238.

4. Y. Benoist and T. Kobayashi, Tempered homogeneous spaces II, In: Dynamics, Geometry, Number Theory: The Impact of Margulis on Modern Mathematics (eds. D. Fisher, D. Kleinbock, and G. Soifer), pp. 213–245, The University of Chicago Press, 2022.

5. Y. Benoist and T. Kobayashi. Tempered homogeneous spaces III. Journal of Lie Theory, **31**, (2021), pp. 833–869.

6. Y. Benoist and T. Kobayashi, Tempered homogeneous spaces IV, preprint. 35 pages, arXiv: 2009.10391. To appea J. Inst. Math. Jussieu.

7. Y. Benoist, Y. Inoue, and T. Kobayashi, Temperedness criterion of the tensor product of parabolic induction for GL_n, preprint. 14 pages. arXiv: 2108.12125.

8. 小林俊行. 緩増加な等質空間 (Tempered Homogeneous Spaces), 2021 年度日本数学会年会函数解析学分科会特別講演アブストラクト, 14 pages.

9. T. Kobayashi and B. Speh. Distinguished

representations of $SO(n+1,1) \times SO(n,1)$, periods and branching laws. In W. Müller, S. W. Shin, and N. Templier, editors, Relative Trace Formulas, Simons Symposia, pages 291–319. Springer, 2021. DOI: 10.1007/978-3-030-68506-5-8.

10. T. Kobayashi, Multiplicity in restricting small representations, preprint. Proc. Japan Acad. **98**, Ser. A Math. Sci., (2022), pp. 19–24.

11. T. Kobayashi, Recent advances in branching problems of representations, To appear in Sugaku Expositions, Amer. Math. Soc. arXiv: 2112.00642.

12. F. Kassel and T. Kobayashi. Spectral analysis on standard locally homogeneous spaces, preprint, 69 pages, ArXiv: 1912.12601.

13. T. Kobayashi, Multiplicity in restricting minimal representations, preprint.

14. T. Kobayashi, Conjectures on reductive homogeneous spaces, 19 pages. To appear in "Mathematics Going Forward", Lecture Notes in Mathematics, vol.2313.

著書:

1. T. Kobayashi and B. Speh. Symmetry Breaking for Representations of Rank One Orthogonal Groups II, Lecture Notes in Mathematics. **2234** Springer, 2018, xv+342 pages. ISBN:978-981-13-2900-5.

2. T. Kobayashi. 分担執筆，解説: リー群の表現論における最近の進展，In: 杉浦光夫『ユニタリ表現入門』, pages 214–242. 東京図書, 2018.

C. 口頭発表

1. Regular Representations on Homogeneous Spaces, (**1.A.**–**1.E.** では講演タイトル，内容は個々に異なるが，大きなテーマとしては同じなので１つにまとめる．) **1.A.** A Foundation of Group-theoretic Analysis on Manifolds. Colloquium di dipartimento. Dipartimento di Matematica, Università di Roma "Tor Vergata" (online), 18 February 2021. **1.B.** Representation Theory of Reductive Groups from Geometric and Analytic Methods (in honour of Simon Gindikin). Kavli IPMU, Japan, 27–28 January 2020; **1.C.** Regular Representations on Homogeneous Spaces. (plenary lecture). International Workshop: Lie Theory and Its Applications in Physics (LT-13). Varna, Blugaria, 17–23 June 2019; **1.D.** Regular Representations on Homogeneous Spaces. (opening lecture). RIMS Workshop: Developments in Representation Theory and Related Topics (organizer: Yoshiki Oshima). RIMS, Kyoto University, 9–12 July 2019; **1.E.** Regular Representations on Homogeneous Spaces. Dynamics of Group Actions: a conference in honor of Yves Benoist. Cetraro, Italy, 27–31 May 2019.

2. Limit Algebras and Tempered Representations. (**2.A.**–**2.D.** では講演タイトル，内容は個々に異なるが，大きなテーマとしては同じなので１つにまとめる．) **2.A.** (opening lecture). RIMS Workshop: Lie Theory, Representation Theory and Related Areas. (online), 10 August 2021. **2.B.** Limit Algebras and Tempered Representations. (plenary opening lecture). XIV. International Workshop: Lie Theory and Its Applications in Physics. Bulgaria (online), 20–26 June 2021. **2.C.** Limit algebras and tempered representation. Lie Groups and Representation Theory Seminar. The University of Tokyo, 15 June 2021. **2.D.** This is What I do: Limit algebras and tempered representations. Representation Theory & Noncommutative Geometry. AIM Research Community (online), 8 April 2021.

3. Global Geometry and Analysis on Locally Symmetric Spaces—Beyond the Riemannian Case. (**3.A.**–**3.K.** では講演タイトル，内容は個々に異なるが，大きなテーマとしては同じなので１つにま

とめる．）**3.A.** Global Analysis of Locally Symmetric Spaces with Indefinite-metric. Colloquium, National University of Singapore. (online), 13 August 2021. **3.B.** Sound of an anti-de Sitter manifold. (opening lecture). Inaugural Day of the French–Kazakhstan school of Mathematics. (online), 25 June 2021. **3.C.** Global Analysis of Locally Symmetric Spaces with Indefinite-metric. Seminar. University of Padova, Italy, 3 June 2019. **3.D.** Global Analysis of Locally Symmetric Spaces with Indefinite-metric. Colloquium. Oklahoma State University, 3 May 2019. **3.E.** Global Analysis of Locally Symmetric Spaces with Indefinite-metric. Colloquium. Yale University, USA, 17 April 2019. **3.F.** Semisimple Symmetric Spaces and Discontinuous Groups: What I Learned from Professor Toshio Oshima. 大島利雄先生古希記念研究集会. Josai University, Tokyo, Japan, 26-27 December 2018. **3.G.** "Geometric Quantization and Applications" M. Vergne 教授記念集会. Luminy, France, 8-12 October 2018. **3.H.** Symposium on Representation Theory 2018. Tottori, Japan, 13-16 November 2018. **3.I.** Colloquium. Hiroshima University, Japan, 6 November 2018. **3.J.** (plenary lecture). The 65th Geometry Symposium. Tohoku University, Sendai, Japan, 28-31 August 2018. **3.K.** Glances at Manifolds: Aleksy Tralle 教授還暦記念研究集会. the Jagiellonian University, Krakow, Poland, 2-6 July 2018.

4. The Kemeny Lectures 2017, I. "Universal sounds" of anti-de Sitter manifolds. The Kemeny lectures, II. Local to global-geometry of symmetric spaces with indefinite-metric, III. Analysis on locally pseudo-Riemannian symmetric spaces. Dartmouth College, USA, 3-5 May 2017.

5. Analysis of minimal representations-an approach to quantize nilpotent orbits. Representation Theory at the Crossroads of Modern Mathematics: Alexandre Kirillov 教授 81 歳記念研究集会. Reims, France, 29 May-2 June 2017.

6. Symmetry Breaking Operators in Conformal Geometry. (**6.A.**–**6.I.** では講演タイトル，内容は個々に異なるが，大きなテーマとしては同じなので１つにまとめる．）**6.A.** Branching Problems and Symmetry Breaking Operators. Geometry, Symmetry and Physics. Yale University, USA, 23 April 2019. **6.B.** Journées SL2R de théorie des représentations et analyse harmonique (Hubert Rubenthaler 教授退官記念研究集会). I.R.M.A., University of Strasbourg, France, 22-23 March 2018. **6.C.** (opening lecture). Joint meeting of X. International Symposium: Quantum Theory and Symmetries and XII. International Workshop: Lie Theory and Its Applications in Physics. Varna, Bulgaria, 19-25 June 2017. **6.D.** (plenary lecture), the XXXV Workshop on Geometric Methods in Physics. Bialowieza, Poland, 2-8 July 2017. **6.E.** Symposium on Representation Theory 2017. Isawa, Yamanashi, Japan, 28 November-1 December 2017. (plenary lecture), 日本. **6.F.** "Geometry and Analysis on Locally Symmetric Spaces with Indefinite-metric—after 145 years of Klein's Erlangen Program". Colloquium. Friedrich-Alexander-Universität Erlangen-Nürnberg, Germany, 25 July 2017. **6.G.** Sophus Lie Seminar. Göttingen, Germany, 30 June-1 July 2017. **6.H.** Harmonic Analysis and the Trace Formula. Oberwolfach, Germany, 21-27 May 2017. **6.I.** AMS Special Session on Harmonic Analysis ('Olafsson 教授 65 歳記念研究集会). Atlanta, USA, 4 January 2017.

7. F-method for Constructing Symmetry Breaking Operators. (**7.A.**–**7.D.** では

講演タイトル，内容は個々に異なるが，大きなテーマとしては同じなので 1 つにまとめる．) **7.A.** Finite Multiplicity Theorems and Real Spherical Varieties. 松本久義氏還暦記念研究集会, (opening lecture) Tokyo, March 27–29, 2019. **7.B.** Abstract Branching Laws for Unitary Highest Weight Modules and Localness Theorem. **7.C.** V. Some Further Perspectives from the General Theory. (**7.B.** と **7.C.** は The 20th Hakuba Workshop on Number Theory in 2017: Automorphic Differential Operators on Siegel Modular Forms (organized by T. Ibukiyama の 5 回連続講演 (分担) の 2 つ). Nagano, Japan, 3-7 September 2017.) **7.D.** Holographic Transform, 20 August, 2021, Workshop on "Actions of Reductive Groups and Global Analysis (Online Tambara), August 17-21, 2021.

8. Branching Laws for Infinite Dimensional Representations of Real Lie Groups; Symmetry Breaking Operators. (**8.A.**–**8.D.** では講演タイトル，内容は個々に異なるが，大きなテーマとしては同じなので 1 つにまとめる．) **8.A.** Joachim Hilgert 教授還暦記念研究集会. Paderborn, Germany, 23-27 July 2018. **8.B.** (plenary lecture). the 32nd International Colloquium on Group Theoretical Methods in Physics (Group32). Czech Technical University, Prague, Czech Republic, 9-13 July 2018. **8.C.** Representation theory, geometry, and quantization: the mathematical legacy of Bertram Kostant. MIT, USA, 28 May-1 June 2018. **8.D.** A Program for Branching Problems in the Representation Theory of Real Reductive Groups: Classification Problem of Symmetry Breaking Operators. Representation Theory inspired by the Langlands Conjectures (organized by B. Speh and P. Trapa), in connection with the AMS-AWM Noether lecture by Birgit Speh. Denver, USA, 17 January 2020.

9. 緩増加な等質空間 (Tempered Homogeneous Spaces). 日本数学会年会函数解析学分科会特別講演 (慶応大学, オンライン), 16 March 2021.

10. Bounded multiplicity in the branching problems of "small" infinite-dimensional representations, 5 October 2021. リー群論·表現論セミナー (オンライン), 東京大学.

D. 講義

1. 数理科学概論 I(文科生) (オンライン): フェルミ推定, 微積分, Taylor 展開, 偏微分, Lagrange の未定乗数法, 近似と概算, 微分方程式の初歩, 多変数関数の積分を講義した. (教養学部文科 1, 2 年生, S セメスター)

2. 数物先端科学 VIII・数学続論 XG (数理大学院・4 年生共通講義, オンライン): Lie Groups and Analytic Approach to Representation Theory (リー群の表現論の解析および幾何的手法) (講義の概要) I explained some basic concepts in representation theory of real reductive Lie groups, and highlighted "multiplicity" of irreducible representations occurring in regular representations. I elucidated an idea of the proof for the following theorem by using simple examples:

geometry $\longleftrightarrow$ representation theory,
real spherical $\longleftrightarrow$ **finite multiplicity**,
spherical $\longleftrightarrow$ **bounded multiplicity**

3. 数学講究 XB (数理科学概説)「不連続群の幾何学と大域解析」, (理学部数学科 4 年生), 2021 年 6 月 8 日.

4. 数学講究 XA, 数学特別講究, 通年: テキスト "Heat Kernels and Dirac Operators" (Berline, Getzler, Vergne) および, "Representations of Semisimple Lie Algebras in the BGG Category $\mathcal{O}$" (Humphreys) (理学部数学科 4 年生)

E. 修士・博士論文

1. (修士) 奥田堯子, Riemann 対称空間上における測地線の簡約部分 Lie 代数への射影に対する有界性—低階数・低次元の場合—

F. 対外研究サービス

1. Kavli IPMU(数物宇宙連携機構), 上席科学研究員併任 (2009.8–2011.5); 主任研究員 (Principal Investigator) 併任 (2011.6–)

[ジャーナルのエディター]

2. Editor in Chief, Japanese Journal of Mathematics （日本数学会–Springer-Nature) (2006–)

3. Editor, International Mathematics Research Notices (Oxford 大学出版) (2002–2021)

4. Editor in Chief, Takagi Booklet, vol. 1–22 (日本数学会) (2006–)

5. Editor, Geometriae Dedicata (Springer) (2000–)

6. Editor, Advances in Pure and Applied Mathematics (de Gruyter) (2008–)

7. Editor, International Journal of Mathematics (World Scientific) (2004–)

8. Editor, Journal of Mathematical Sciences, The University of Tokyo (2007–)

9. Editor, Kyoto Journal of Mathematics (2010–)

10. Editor, Representation Theory (アメリカ数学会) (2015–2019)

11. Editor, AMS Translation Series (アメリカ数学会) (2016–)

12. Editor, Tunijian Journal of Mathematics (2017–)

13. Editor, Special Issue in commemoration of Professor Kunihiko Kodaira's centennial birthday (J. Math. Sciences, The University of Tokyo).

14. Editor, Special Issue in honor of Professor Masaki Kashiwara's 70th birthday (Publ. RIMS) 2017–2021.

15. Chief Editor, Mikio Sato's Collected Papers, (Springer-Nature).

16. 共立出版,『共立講座 数学探検（全 18 巻)』,『共立講座 数学の魅力（全 14 巻＋別巻 1)』,『共立講座 数学の輝き（全 40 巻予定)』の 3 シリーズ編集委員

17. 編集委員, 数学の現在 i, e, π, (with 斎藤毅, 河東泰之), 東京大学出版会.

[学会・他大学の委員など]

18. ある国際数学者賞の授賞委員会: Prize Committee (International Prize, 数学部門, 国外) 2018.

19. ある国際賞の授賞委員会: Prize Committee (mathematics), 2019–2020.

20. ある国際委員会の責任者, 2019–2022.

21. ある国際賞の授賞委員会: Prize Committee (mathematics), 2020–2021.

22. 国内審査委員: Prize Committee 日本数学会春季賞・秋季賞他 (anonymous) (various years)

23. 京都大学数理解析研究所運営委員 (2015–2017; 2017–2019)

24. 京都大学数理解析研究所専門委員 (2007–2009; 2009–2011; 2015–2017; 2017–2019; 2021–)

25. 科学研究費等の審査委員: 日本 (JSPS), 米国 (NSF-AMS), EU, ドイツ, ルクセンブルク, 中華人民共和国・香港 (various years)

26. OIST Advisory Board (2021–).

[国際研究集会のオーガナイザーなど]

27. オーガナイザー, Summer School on Representation Theory, リー群の群作用と大域解析に関するセミナー, (virtual 玉原国際セミナーハウス), August 17–21, 2021(オンライン).

28. オーガナイザー, Integral Geometry, Representation Theory and Complex Analysis, Kavli Institute for the Physics and Mathematics of the Universe, 27–28 January 2020.

29. オーガナイザー, 高木レクチャー, 第 24 回 (東京大学 IPMU, 2019 年 12 月); 第 23 回 (京都大学数理研, 2019 年 6 月) (with Y. Kawahigashi, T. Kumagai, H. Nakajima, K. Ono and T. Saito).

30. オーガナイザー, Summer School on Representation Theory, リー群の群作用と大域解析に関するセミナー, (virtual 玉原国際セミナーハウス), 18–22 August 2020 (オンライン).

31. オーガナイザー, Summer School on Representation Theory, リー群の群作用と大域解

析に関するセミナー，玉原国際セミナーハウス，20–24 August 2019.

32. オーガナイザー, Summer School on Representation Theory, リー群の群作用と大域解析に関するセミナー，玉原国際セミナーハウス，19–23 August 2018.

33. オーガナイザー, Summer School on Representation Theory, リー群の群作用と大域解析に関するセミナー，玉原国際セミナーハウス，16–20 August 2017.

34. オーガナイザー, 高木レクチャー, 第 19 回 (京都大学数理研，2017 年 7 月)，第 20 回 (東京大学，2017 年 11 月) 第 21 回 (京都大学数理研, 2018 年 6 月)，第 22 回 (東京大学, 2018 年 11 月) (with Y. Kawahigashi, H. Nakajima, K. Ono and T. Saito)

35. オーガナイザー, リー群論・表現論セミナー (2007–present 東大; 2003–2007 RIMS; 1987–2001 東大)

G. 受賞

1. 日本数学会出版賞 (2019) 『数学の現在 i, e, π』東京大学出版会，(斎藤毅氏，河東泰之氏との共同受賞).

2. アメリカ数学会フェロー (2017) 「簡約リー群の構造論と表現論に対する貢献」(Contribution to Structure Theory and Representation Theory of Reductive Lie groups).

3. [学生の受賞] 東京大学学位記授与式における総代, 甘中一輝 (2020 年度 (2021 年 3 月), 総代・答辞),
田森宥好 (2019 年度 (2020 年 3 月)，総代).

H. 海外からのビジター

連携併任講座