SINGULAR UNITARY REPRESENTATIONS AND DISCRETE SERIES FOR INDEFINITE STIEFEL MANIFOLDS $U(p,q;\mathbb{F})/U(p-m,q;\mathbb{F})$

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Abstract

This paper treats the relatively singular part of the unitary dual of pseudo-orthogonal groups over $\mathbb{F} = \mathbb{R}$, $\mathbb{C}$ and $\mathbb{H}$. These representations arise from discrete series for indefinite Stiefel manifolds $U(p,q;\mathbb{F})/U(p-m,q;\mathbb{F})$ $(2m \le p)$. Thanks to the duality theorem between $\mathcal{D}$-module construction and Zuckerman's derived functor modules (ZDF-modules), these discrete series are naturally described in terms of ZDF-modules with possibly singular parameters. Some techniques including a new K-type formula are offered to find the explicit condition deciding whether the corresponding ZDF-modules $\mathcal{R}^S_{\mathfrak{q}}(\mathbb{C}_\lambda)$ vanish or not. We also investigate the irreducibility and pairwise inequivalence among these ZDF-modules. Although our concern is limited to the discrete series, our approach is purely algebraic and applicable to a less special setting. It is an interesting phenomenon that our discrete series sometimes give a sharper condition for unitarizability of ZDF-modules than those given by Vogan (1984) algebraically. This phenomenon does not occur in the case of discrete series for group manifolds or semisimple symmetric spaces.

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