Integral Geometry, Representation Theory and Complex Analysis

Kavli Institute for the Physics and Mathematics of the Universe

27–28 January 2020

ABSTRACT

Leticia Barchini (Oklahoma University)
Cells of Harish-Chandra Modules

The Grothendieck group of Harish-Chandra modules having the same infinitesimal character as the trivial representation affords an action of the Weyl group via the coherent continuation representation. The equivalence relation generated by the condition of two irreducible modules having a non-split extension defines equivalence classes of modules known as blocks. The span of representations on each block is preserved by the coherent continuation action. Furthermore, each block is partitioned into Harish-Chandra cells. The elements of a Harish-Chandra index a basis of a subquotient of the full coherent continuation representation; the Harish-Chandra cells representations. Harish-Chandra cells representations have been the focus of deep study by Barbasch and Vogan in the 1980's. They are important tools in the study of representations of real reductive groups and their geometric invariants. It is known that HC cells for $\mathrm{Sp}(p,q), \mathrm{SO}^*(2n), \mathrm{SU}(p,q)$ are indexed by "real" nilpotent orbits. These orbits are the associated variety of the modules in the corresponding cells. In this talk I will consider the coherent continuation action on the Grothendieck group for $\mathrm{Sp}(2n, \mathbf{R})$ and I will discuss a parametrization of HC cells in terms of geometric objects.

* * * * *

Joseph Bernstein (Tel Aviv University and The University of Tokyo)
Geometric Realization of Harish-Chandra Modules

Let (G, K) be a semisimple reductive pair over $\mathbf{C}$, i.e. G is a reductive group and $K = G^\gamma$ be the subgroup of fixed points of some involutive automorphism $\gamma : G \to G$.

In representation theory of real reductive groups important role is played by the category $\mathcal{M}(\mathfrak{g}, K)$ of $(\mathfrak{g}, K)$ modules and its subcategory $\mathcal{H}(\mathfrak{g}, K)$ of Harish-Chandra modules.

In my talk I will describe a construction that gives a geometric description of this category. Namely I will construct a smooth variety T with an action of the group G, a G-eqivariant sheaf of associative D-algebras A on T and a G-invariant subvariety $X \subset T$ such that the category $\mathcal{M}(\mathfrak{g}, K)$ is equivalent to the category of G-equivariant A-modules supported on X.

This allows to interpret Harish-Chandra modules as some geometric objects and study them using geometry.

I will discuss some applications of this interpretation.

* * * * *

Laura Geatti (University of Roma)
The Adapted Hyper-Kähler Structure on the Tangent Bundle of a Hermitian Symmetric Space

The cotangent bundle of a compact Hermitian symmetric space $X = G/K$ (a tubular neighbourhood of the zero section, in the non-compact case) carries a unique G-invariant hyper-Kähler structure compatible with the Kähler structure of X and the canonical complex symplectic form of T^*X (cf. Biquard and Gauduchon).

The tangent bundle TX, which is isomorphic to T^*X, carries a canonical complex structure J, the so called "adapted complex structure", and admits a unique G-invariant hyper-Kähler structure compatible with the Kähler structure of X and with J. The two hyper-Kähler structures are related by a G-equivariant fiber preserving diffeomorphism of TX (cf. Dancer and Szöke).

The fact that the domain of existence of J in TX is biholomorphic to a G-invariant domain in the complex homogeneous space $G^{\mathbf{C}}/K^{\mathbf{C}}$ enables us to use Lie theoretical tools and moment map techniques to explicitly compute the various quantities of the "adapted hyper-Kähler structure". This is joint work with Andrea Iannuzzi.

* * * * *

Simon Gindikin (Rutgers University)
Universal Nature of the Horospherical Transform in Symmetric Spaces

Using the example of the rank one case we discuss that the horospherical transform is a universal method of harmonic analyses on symmetric spaces which works in all cases, including the compact and pseudo Riemannian ones. For discrete series of representations we need to modify the classical horospherical transform by considering its complexified version—the horospherical Cauchy transform—which acts in holomorphic functions or Cauchy–Riemann cohomology. Another important circumstance which we discuss is that the problem of the inversion of the horospherical transform on symmetric spaces is homological to the similar problem for flat models of these symmetric spaces.

* * * * *

Mikhail Kapranov (Kavli IPMU)
Bivariant Approach to Perverse Sheaves

Intersection homology can be seen as situated in the middle between cohomology and homology. Similarly, perverse sheaves can be seen as situated in the middle between sheaves and cosheaves, i.e., covariant and contravariant data on cells of a cell decomposition. In the talk, I present a point of view on perverse sheaves that makes this point of view literally true in some examples. More precisely, I will present a description of perverse sheaves as data which are covariant in one half of the directions and contravariant in the other half. Joint work with V. Schechtman.

* * * * *

Toshiyuki Kobayashi (The University of Tokyo and Kavli IPMU)
Regular Representations on Homogeneous Spaces

I plan to discuss some basic questions about regular representations on X acted algebraically by real reductive groups G.

1. (function spaces) Does the group G have a "good control" on the space $C(X)$ of function on X?
2. (L^2 theory) What can we say about "spectrum" for $L^2(X)$?

We highlight "multiplicities" and "temperdness" for these questions, and give their geometric criteria.

If time permits, I will mention some applications to branching problems for restriction of infinite-dimensional representations.

* * * * *

Michael Pevzner (University of Reims)
From Symmetry Breaking toward Holographic Transform in Representation Theory

We shall present the idea of symmetry breaking transform in the framework of branching rules for infinite dimensional representations of reductive Lie groups and will focus on the dual notion of holographic transform by providing a series of concrete examples.

* * * * *

Taito Tauchi (The University of Tokyo)
Relationship between Orbit Decomposition on the Flag Varieties and Multiplicities of Induced Representations

Let G be a real reductive Lie group and H a closed subgroup. T. Kobayashi and T. Oshima established a finiteness criterion of multiplicities of irreducible G-modules occurring in the regular representation $C^\infty(G/H)$ by a geometric condition, referred to as real sphericity, namely, H has an open orbit on the real flag variety G/P. In this lecture, we discuss a refinement of their theorem by replacing a minimal parabolic subgroup P with a general parabolic subgroup Q of G, where a careful analysis is required because the finiteness of the number of H-orbits on the partial flag variety G/Q is not equivalent to the existence of H-open orbit on G/Q.