FMSP Lectures

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"K-THEORY AND THE DIRAC OPERATOR"

Lecture 1. WHAT IS K-THEORY AND WHAT IS IT GOOD FOR?
October 22 (Mon), 15:00 ～ 16:30　Room 123

This talk will consist of four points.
　　1. The basic definition of K-theory
　　2. A brief history of K-theory
　　3. Algebraic versus topological K-theory
　　4. The unity of K-theory

Lecture 2.　THE DIRAC OPERATOR
October 24 (Wed), 15:00 ～ 16:30　Room 123

The Dirac operator of R^n will be defined. This is a first order elliptic differential operator with constant coefficients.

Next, the class of differentiable manifolds which come equipped with an order one differential operator which (at the symbol level)is locally isomorphic to the Dirac operator of R^n will be considered. These are the Spin-c manifolds.

Spin-c is slightly stronger than oriented, so Spin-c can be viewed as "oriented plus epsilon". Most of the oriented manifolds that occur in practice are Spin-c. The Dirac operator of a closed Spin-c manifold is the basic example for the Hirzebruch-Riemann-Roch theorem and the Atiyah-Singer index theorem.

Lecture 3.   THE RIEMANN-ROCH THEOREM
October 29 (Mon), 15:00 ～ 16:30   Room 117

Topics in this talk :
1. Classical Riemann-Roch
2. Hirzebruch-Riemann-Roch (HRR)
3. Grothendieck-Riemann-Roch (GRR)
4. RR for possibly singular varieties (Baum-Fulton-MacPherson)

Lecture 4.   BEYOND ELLIPTICITY or K-HOMOLOGY AND INDEX THEORY ON CONTACT MANIFOLDS
October 31 (Wed), 15:00 ～ 16:30   Room 122

K-homology is the dual theory to K-theory. The BD (Baum-Douglas) isomorphism of Atiyah-Kasparov K-homology and K-cycle K-homology provides a framework within which the Atiyah-Singer index theorem can be extended to certain differential operators which are hypoelliptic but not elliptic. This talk will consider such a class of differential operators on compact contact manifolds. These operators have been studied by a number of mathematicians (e.g. C.Epstein and R.Melrose).

Operators with similar analytical properties have also been studied (e.g. by Alain Connes and Henri Moscovici ⋯ also Michel Hilsum and Georges Skandalis). Working within the BD framework, the index problem will be solved for these differential operators on compact contact manifolds.

This is joint work with Erik van Erp.