Lie Groups and Representation Theory Seminar at the University of Tokyo

リー群論・表現論セミナー

DATE October 14 (Tue), 2008, 16:30–18:00

- PLACE Room 126, Graduate School of Mathematical Sciences
- SPEAKER Jan Moellers (Paderborn University)
 - TITLE The Dirichlet-to-Neumann map as a pseudodifferential operator
- Both Dirichlet and Neumann boundary conditions for the Abstract Laplace equation are of fundamental importance in Mathematics and Physics. Given a compact connected Riemannian manifold M with boundary ∂M the Dirichlet-to-Neumann operator Λ_q maps Dirichlet boundary data f to the corresponding Neumann boundary data $\Lambda_q f = (\partial_{\nu} u)|_{\partial M}$ where u denotes the unique solution to the Dirichlet problem $\triangle_g u = 0$ in M, $u|_{\partial M} = f$. The main statement is that this operator is a first order elliptic pseudodifferential operator on the boundary ∂M . We will first give a brief overview of how to define the Dirichlet-to-Neumann operator as a map $\Lambda_q: H^{1/2}(\partial M) \longrightarrow$ $H^{-1/2}(\partial M)$ between Sobolev spaces. In order to show that it is actually a pseudodifferential operator we introduce tangential pseudodifferential operators. This allows us to derive a microlocal factorization of the Laplacian near boundary points. Together with a regularity statement for the heat equation this will finally give the main result.