Lie Group and Representation Theory Seminar

Date: November 12 (Wed) , 2003, 18:00–19:00
Place: RIMS 402
Speaker: Christof Geiss, (Ciudad Universitaria, Mexico)
Title: Semicanonical bases and preprojective algebras

Abstract:

This is a report on joint work with J. Schröer and B. Leclerc.
Let $\mathfrak{g}$ be a simple Lie algebra of type $\mathsf{A}, \mathsf{D}, \mathsf{E}$ and $\mathfrak{n}$ a maximal nilpotent subalgebra of $\mathfrak{g}$. Moreover, let N be a maximal unipotent subgroup of a simple Lie group with Lie algebra $\mathfrak{g}$. Finally, let Π denote the corresponding preprojective algebra.

Lusztig's semicanonical basis $\mathcal{S}$ of $U(\mathfrak{n})$ is parametrized by irreducible components of the corresponding nilpotent varieties $\mathrm{mod}(\Pi, \mathbf{d})$. The dual $\mathcal{S}^*$ is a basis of $\mathbb{C}[N]$.

We can show: For two elements of $\mathcal{S}^*$ holds $b_C \cdot b_D \in \mathcal{S}^*$ if for the corresponding irreducible components holds $\mathrm{ext}^1_\Pi(C, D) = 0$.

On the other hand, the dual canonical basis $\mathcal{B}^*_q$ of $U_q(\mathfrak{n})$ specializes for $q = 1$ to a basis $\mathcal{B}$. We can show, that $\mathcal{S}$ and $\mathcal{B}$ have many elements in common, but $\mathcal{B}$ and $\mathcal{S}$ coincide only if Π is representation finite, i.e. in the cases $\mathsf{A}_{2,3,4}$. This explains the multiplicative properties of the dual canonical basis observed previously in these cases.

On the other hand it gives us a good control over the dual semicanonical basis in the cases A_5 and D_4, i.e. when Π is tame, since we have in this case a precise combinatorial description of the irreducible components of $\mathrm{mod}(\Pi, \mathbf{d})$ in terms of indecomposable components. This is closely related to an elliptic root system of type $\mathsf{E}_8^{(1,1)}$ resp. $\mathsf{E}_6^{(1,1)}$.

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