Harmonic analysis on the Iwahori–Hecke algebra

Yuval Z. Flicker

Abstract. These are purely expository notes of Opdam's analysis [O1] of the trace form $\tau(f) = f(e)$ on the Hecke algebra $H = C_c(I \backslash G / I)$ of compactly supported functions f on a connected reductive split p-adic group G which are biinvariant under an Iwahori subgroup I, extending Macdonald's work. We attempt to give details of the proofs, and choose notations which seem to us more standard. Many objects of harmonic analysis are met: principal series, Macdonald's spherical forms, trace forms, Bernstein forms. The latter were introduced by Opdam under the name Eisenstein series for H. The idea of the proof is that the last two linear forms are proportional, and the proportionality constant is computed by projection to Macdonald's spherical forms. Crucial use is made of Bernstein's presentation of the Iwahori–Hecke algebra by means of generators and relations, as an extension of a finite dimensional algebra by a large commutative subalgebra. We give a complete proof of this using the universal unramified principal series right H-module $M = C_c(A(O)N \backslash G / I)$ to develop a theory of intertwining operators algebraically.

Keywords and phrases: Iwahori–Hecke algebra, Bernstein presentation, intertwining operators, trace, generating function, Bernstein forms, trace forms, Macdonald's spherical forms

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Y. Z. FLICKER
The Ohio State University, Columbus, Ohio 43210, USA and
Ariel University, Ariel 40700, Israel
(e-mail: yzflicker@gmail.com)