Japan. J. Math. 7, 135–166 (2012)
DOI: 10.1007/s11537-012-1220-9

JAPANESE JOURNAL OF MATHEMATICS
SINCE 1924

Introduction to random walks on homogeneous spaces*

Yves Benoist · Jean-François Quint

Received: 30 May 2012 / Revised: 4 September 2012 / Accepted: 12 September 2012
Published online: 17 November 2012

Communicated by: Toshiyuki Kobayashi

Abstract. Let a_0 and a_1 be two matrices in $\mathrm{SL}(2,\mathbb{Z})$ which span a non-solvable group. Let x_0 be an irrational point on the torus $\mathbb{T}^2$. We toss a_0 or a_1, apply it to x_0, get another irrational point x_1, do it again to x_1, get a point x_2, and again. This random trajectory is equidistributed on the torus. This phenomenon is quite general on any finite volume homogeneous space.

Keywords and phrases: Lie groups, discrete subgroups, homogeneous dynamics, random walk

Mathematics Subject Classification (2010): 22E40, 37C85, 60J05

---

* This article is based on the 10th Takagi Lectures that the author delivered at Research Institute for Mathematical Sciences, Kyoto University on May 26, 2012.

Y. BENOIST
Centre national de la recherche scientifique–Département de Mathématiques, Université Paris-Sud 11, Bâtiment 425, F-91405 Orsay Cedex, France
(e-mail: yves.benoist@math.u-psud.fr)

J.-F. QUINT
Centre national de la recherche scientifique–Université Paris-Nord, LAGA, 99, avenue Jean-Baptiste Clément, 93430 Villetaneuse, France
(e-mail: quint@math.univ-paris13.fr)