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The BC-system and L-functions*

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Abstract. In these lectures we survey some relations between L-functions and the BC-system, including new results obtained in collaboration with C. Consani. For each prime p and embedding σ of the multiplicative group of an algebraic closure of $\mathbb{F}_p$ as complex roots of unity, we construct a p-adic indecomposable representation π_σ of the integral BC-system. This construction is done using the identification of the big Witt ring of $\bar{\mathbb{F}}_p$ and by implementing the Artin–Hasse exponentials. The obtained representations are the p-adic analogues of the complex, extremal KMS_∞ states of the BC-system. We use the theory of p-adic L-functions to determine the partition function. Together with the analogue of the Witt construction in characteristic one, these results provide further evidence towards the construction of an analogue, for the global field of rational numbers, of the curve which provides the geometric support for the arithmetic of function fields.

Keywords and phrases: BC-system, Witt rings, adèle class space

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