

Arithmetic applications of the Langlands program^{*}

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Abstract. This expository article is an introduction to the Langlands functoriality conjectures and their applications to the arithmetic of representations of Galois groups of number fields. Thanks to the work of a great many people, the stable trace formula is now largely established in a version adequate for proving Langlands functoriality in the setting of endoscopy. These developments are discussed in the first two sections of the article. The final section describes the compatible families of ℓ -adic Galois representations that can be attached to automorphic forms with the help of Shimura varieties. To illustrate the relevance of Langlands functoriality to number theory, the article concludes with a description of the Sato–Tate conjecture for elliptic modular forms, recently proved in joint work of Barnet-Lamb, Geraghty, Taylor, and the author.

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