

## On the mathematics of emergence<sup>\*</sup>

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**Abstract.** We describe a setting where convergence to consensus in a population of autonomous agents can be formally addressed and prove some general results establishing conditions under which such convergence occurs. Both continuous and discrete time are considered and a number of particular examples, notably the way in which a population of animals move together, are considered as particular instances of our setting.

*Keywords and phrases:* emergence, flocking, consensus reaching

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