

Orbifolds of lattice vertex algebras

Bojko Bakalov · Jason Elsinger · Victor G. Kac · Ivan Todorov

Received: 21 October 2022 / Accepted: 28 February 2023

Published online: 14 June 2023

© The Mathematical Society of Japan and Springer Nature Japan KK, part of Springer Nature 2023

Communicated by: Yasuyuki Kawahigashi

Abstract. To a positive-definite even lattice Q , one can associate the lattice vertex algebra V_Q , and any automorphism σ of Q lifts to an automorphism of V_Q . In this paper, we investigate the orbifold vertex algebra V_Q^σ , which consists of the elements of V_Q fixed under σ , in the case when σ has prime order. We describe explicitly the irreducible V_Q^σ -modules, compute their characters, and determine the modular transformations of characters. As an application, we find the asymptotic and quantum dimensions of all irreducible V_Q^σ -modules. We consider in detail the cases when the order of σ is 2 or 3, as well as the case of permutation orbifolds.

Keywords and phrases: regular vertex algebra, lattice vertex algebra, theta function, modular transformation, orbifold algebra, twisted module

Mathematics Subject Classification (2020): Primary 17B69; Secondary 81R10

B. BAKALOV

Department of Mathematics, North Carolina State University Raleigh, NC 27695, USA
(e-mail: bojko_bakalov@ncsu.edu)

J. ELSINGER

Department of Mathematics, Florida Southern College Lakeland, FL 33801, USA
(e-mail: jelsinger@flsouthern.edu)

V.G. KAC

Department of Mathematics, MIT, Cambridge, MA 02139, USA
(e-mail: kac@math.mit.edu)

I. TODOROV

INRNE, Bulgarian Academy of Sciences, Tsarigradsko Chaussee 72, BG-1784 Sofia, Bulgaria
(e-mail: ivbortodorov@gmail.com)

Contents

1. Introduction	170
2. Vertex algebras and their twisted modules	171
2.1. Conformal vertex algebras	171
2.2. Twisted representations of vertex algebras	173
2.3. Regular vertex algebras	174
2.4. Theta functions and transformation laws	177
3. Twisted representations of lattice vertex algebras	182
3.1. Lattice vertex algebras	182
3.2. Twisted Heisenberg algebra	184
3.3. The groups G_σ and G_σ^\perp	185
3.4. The G_σ -modules $W(\mu, \zeta)$	188
3.5. The σ -twisted V_Q -modules $M(\mu, \zeta)$	191
3.6. L_0^{tw} -action on the V_Q -module $M(\mu, \zeta)$	193
3.7. The sublattice $\bar{Q} \subset Q$	196
3.8. The subgroup $\Gamma_{\sigma, \mu, \zeta}$	201
4. Modified characters of twisted V_Q -modules	203
4.1. Calculating the trace over \mathcal{F}_σ	203
4.2. The sublattice R_\perp and a basis for $W(\mu, \zeta)$	206
4.3. Calculating the trace over the G_σ -module $W(\mu, \zeta)$	208
4.4. Calculating the trace over the untwisted V_Q -module $V_{\lambda+Q}$	212
4.5. Irreducible orbifold modules and their characters	215
5. Transformation laws for modified characters of twisted V_Q -modules	220
5.1. Transformation laws for modified characters of \mathcal{F}_σ	220
5.2. Transformation laws for modified characters of $W(\lambda)$ and $W(\mu, \zeta)$	227
5.3. Transformation laws for modified characters of $V_{\lambda+M}$ and $M(\mu, \zeta)$	233
5.4. Orbifold modules and transformation laws for orbifold characters	235
6. Examples in order 2	240
6.1. The irreducible characters of twisted type in the general setting	240
6.2. The irreducible characters and S -matrix of a \mathbb{Z}_2 -orbifold using the root lattice A_2	244
6.3. The irreducible characters and S -matrix of a \mathbb{Z}_2 -orbifold using the root lattice A_3	247
7. Examples in order 3	254
7.1. The irreducible characters of twisted type in the general setting	254
7.2. The \mathbb{Z}_3 -orbifold using the root lattice D_4	256
8. Permutation orbifolds	259
8.1. The irreducible characters in the general setting	259
8.2. The case when $p = 3$	270