Japan. J. Math. 17, 63–116 (2022)
DOI: 10.1007/s11537-021-2134-1

JAPANESE JOURNAL OF MATHEMATICS
SINCE 1924

On Lax operators

Alberto De Sole · Victor G. Kac · Daniele Valeri

Received: 15 July 2021 / Revised: 29 September 2021 / Accepted: 8 October 2021
Published online: 10 December 2021
© The Mathematical Society of Japan and Springer Japan KK, part of Springer Nature 2021

Communicated by: Yasuyuki Kawahigashi

Abstract. We define a Lax operator as a monic pseudodifferential operator $L(\partial)$ of order $N \geq 1$, such that the Lax equations $\frac{\partial L(\partial)}{\partial t_k} = [(L^{\frac{k}{N}}(\partial))_+, L(\partial)]$ are consistent and non-zero for infinitely many positive integers k. Consistency of an equation means that its flow is defined by an evolutionary vector field. In the present paper we demonstrate that the traditional theory of the KP and the N-th KdV hierarchies holds for arbitrary scalar Lax operators.

Keywords and phrases: Lax equation, Lax operator, KP hierarchy, N-th KdV hierarchy, wave function, tau-function

Mathematics Subject Classification (2020): Primary 17B80; Secondary 37K10, 37K30

Contents

1. Introduction 64
2. Algebraic setup 69

A.De Sole
Dipartimento di Matematica, Sapienza Università di Roma, P.le Aldo Moro 2, 00185 Rome, Italy
(e-mail: desole@mat.uniroma1.it)

V.G. Kac
Dept of Mathematics, MIT, 77 Massachusetts Avenue, Cambridge, MA 02139, USA
(e-mail: kac@math.mit.edu)

D. Valeri
School of Mathematics & Statistics, University of Glasgow, G12 8QQ Glasgow, UK
(e-mail: daniele.valeri@glasgow.ac.uk)

3. Lax operators and hierarchies of Lax equations 78
4. Lax equations and Zakharov–Shabat equations 89
5. Lax and Sato equations, linear problem and bilinear equation 91
6. Wave functions and tau-functions 97