Recent results on the Kobayashi and Green–Griffiths–Lang conjectures*

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in celebration of the 100th anniversary of K. Kodaira's birth*

Abstract. The study of entire holomorphic curves contained in projective algebraic varieties is intimately related to fascinating questions of geometry and number theory—especially through the concepts of curvature and positivity which are central themes in Kodaira's contributions to mathematics. The aim of these lectures is to present recent results concerning the geometric side of the problem. The Green–Griffiths–Lang conjecture stipulates that for every projective variety X of general type over $\mathbb{C}$, there exists a proper algebraic subvariety Y of X containing all entire curves $f : \mathbb{C} \to X$. Using the formalism of directed varieties and jet bundles, we show that this assertion holds true in case X satisfies a strong general type condition that is related to a certain jet-semi-stability property of the tangent bundle T_X. It is possible to exploit similar techniques to investigate a famous conjecture of Shoshichi Kobayashi (1970), according to which a generic algebraic hypersurface of dimension n and of sufficiently large degree $d \geqslant d_n$ in the complex projective space $\mathbb{P}^{n+1}$ is hyperbolic: in the early 2000's, Yum-Tong Siu proposed a strategy that led in 2015 to a proof based on a clever use of slanted vector fields on jet spaces, combined with Nevanlinna theory arguments. In 2016, the conjecture has been settled in a different way by Damian Brotbek, making a more direct use of Wronskian differential operators and associated multiplier ideals; shortly afterwards, Ya Deng showed how the proof could be modified to yield

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an explicit value of d_n. We give here a short proof based on a substantial simplification of their ideas, producing a bound very similar to Deng's original estimate, namely $d_n = \lfloor \frac{1}{3}(en)^{2n+2} \rfloor$.