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Singularities in mixed characteristic. The perfectoid approach*

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Abstract. The homological conjectures, which date back to Peskine, Szpiro and Hochster in the late 60's, make fundamental predictions about syzygies and intersection problems in commutative algebra. They were settled long ago in the presence of a base field and led to tight closure theory, a powerful tool to investigate singularities in characteristic p.

Recently, perfectoid techniques coming from p-adic Hodge theory have allowed us to get rid of any base field; this solves the direct summand conjecture and establishes the existence and weak functoriality of big Cohen–Macaulay algebras, which solve in turn the homological conjectures in general. This also opens the way to the study of singularities in mixed characteristic.

We sketch a broad outline of this story, taking lastly a glimpse at ongoing work by L. Ma and K. Schwede, which shows how such a study could build a bridge between singularity theory in characteristic p and in characteristic 0.

Keywords and phrases: singularities, homological conjectures, big Cohen–Macaulay algebras, perfectoid spaces

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