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# Conformal embeddings of affine vertex algebras in minimal $W$-algebras II: decompositions

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**Abstract.** We present methods for computing the explicit decomposition of the minimal simple affine $W$-algebra $W_k(\mathfrak{g},\theta)$ as a module for its maximal affine subalgebra $\mathcal{V}_k(\mathfrak{g}^\natural)$ at a conformal level $k$, that is, whenever the Virasoro vectors of $W_k(\mathfrak{g},\theta)$ and $\mathcal{V}_k(\mathfrak{g}^\natural)$ coincide. A particular emphasis is given on the application of affine fusion rules to the determination of branching rules. In almost all cases when $\mathfrak{g}^\natural$ is a semisimple Lie algebra, we show that, for a suitable conformal level $k$, $W_k(\mathfrak{g},\theta)$ is isomorphic to an extension of $\mathcal{V}_k(\mathfrak{g}^\natural)$ by its simple module. We are able to prove that in certain cases $W_k(\mathfrak{g},\theta)$ is a simple current extension of $\mathcal{V}_k(\mathfrak{g}^\natural)$. In order to analyze more complicated non simple current extensions at conformal levels, we present an explicit realization of the simple $W$-algebra $W_k(sl(4),\theta)$ at $k=-8/3$. We prove, as conjectured in [3], that $W_k(sl(4),\theta)$ is isomorphic to the vertex algebra $\mathscr{R}^{(3)}$, and construct infinitely many singular vectors using screening operators. We also construct a new family of simple current

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modules for the vertex algebra $V_k(sl(n))$ at certain admissible levels and for $V_k(sl(m|n)), m \neq n, m, n \geq 1$ at arbitrary levels.

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