

A sufficient condition for a rational differential operator to generate an integrable system

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Abstract. For a rational differential operator $L = AB^{-1}$, the Lenard–Magri scheme of integrability is a sequence of functions $F_n, n \geq 0$, such that (1) $B(F_{n+1}) = A(F_n)$ for all $n \geq 0$ and (2) the functions $B(F_n)$ pairwise commute. We show that, assuming that property (1) holds and that the set of differential orders of $B(F_n)$ is unbounded, property (2) holds if and only if L belongs to a class of rational operators that we call integrable. If we assume moreover that the rational operator L is weakly non-local and preserves a certain splitting of the algebra of functions into even and odd parts, we show that one can always find such a sequence (F_n) starting from any function in $\text{Ker } B$. This result gives some insight in the mechanism of recursion operators, which encode the hierarchies of the corresponding integrable equations.

Keywords and phrases: integrable systems, Lenard–Magri scheme of integrability, rational pseudo-differential operators, symmetries

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