

## Perfectoid Shimura varieties<sup>★</sup>

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Received: 2 June 2015 / Revised: 2 September 2015 / Accepted: 10 September 2015

Published online: 17 November 2015

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Communicated by: Takeshi Saito

**Abstract.** This note explains some of the author’s work on understanding the torsion appearing in the cohomology of locally symmetric spaces such as arithmetic hyperbolic 3-manifolds.

The key technical tool was a theory of Shimura varieties with infinite level at  $p$ : As  $p$ -adic analytic spaces, they are perfectoid, and admit a new kind of period map, called the Hodge–Tate period map, towards the flag variety. Moreover, the (semisimple) automorphic vector bundles come via pullback along the Hodge–Tate period map from the flag variety.

In the case of the Siegel moduli space, the situation is fully analyzed in [?]. We explain the conjectural picture for a general Shimura variety.

*Keywords and phrases:* Shimura varieties, Galois representations, perfectoid spaces

*Mathematics Subject Classification (2010):* 14G35, 11F03, 11F80, 14G22

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<sup>★</sup> This article is based on the 14th Takagi Lectures that the author delivered at the University of Tokyo on November 15 and 16, 2014.

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