

## Miscellany on traces in $\ell$ -adic cohomology: a survey

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*To Tetsuji Shioda, on the occasion of his 65th birthday*

**Abstract.** We discuss classical questions concerning traces of elements of Galois groups or correspondences in  $\ell$ -adic cohomology, mostly over finite or local fields, such as rationality and independence of  $\ell$ , integrality, congruences modulo powers of  $\ell$  or  $p$ . We report on the progress that has been made on this topic during the past ten years.

*Keywords and phrases:*  $\ell$ -adic cohomology, independence of  $\ell$ , Grothendieck's trace formula, Lefschetz trace formula, zeta functions over finite fields, Euler–Poincaré characteristic, Betti number, Bloch's conductor conjecture, intersection cohomology, Grothendieck's six operations, intermediate extension, Weil conjectures, Hodge polygon, Newton polygon, crystalline cohomology, Hodge filtration, coniveau filtration, alteration, Fano variety, rationally connected, Weil group, Swan conductor, wild ramification, Brauer trace, log scheme, logarithmic differential forms, Čebotarev's density theorem, semisimple group, Fatou's lemma

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