1) ℓ-adic Riemann-Roch formula j.w. K. Kato

$f : U \to V$ morphism of sep sch of f.t / k.

$$\begin{array}{ccc} f_! : K_0(U, \bar{\mathbb{Q}}_\ell) & \longrightarrow & K_0(V, \bar{\mathbb{Q}}_\ell) \\ \downarrow & \circlearrowleft & \downarrow \\ ? & \longrightarrow & ? \end{array}$$

Groth. gp of ℓ-adic const. sheaves

$k = \mathbb{C}$ MacPherson. Singular homology, Chern class

$k = \bar{k}$ char $p > 0$. $V = \operatorname{Spec} k$ $K_0(V, \bar{\mathbb{Q}}_\ell) = \mathbb{Z}$.

U smooth. $\mathcal{F}$ smooth

$$\chi_c(U_{\bar{k}}, \mathcal{F}) = \operatorname{rk} \mathcal{F} \cdot \chi_c(U_{\bar{k}}, \mathbb{Q}_\ell) - \deg Sw_U \mathcal{F}$$

$\Uparrow$ $CH_0(X \setminus U)_{\mathbb{Q}}$

$\Uparrow$ compactification of X

gen'n of G-O-S. formula

$k = K$ p-adic field. c.d.v.f char $= 0$ F res. field perfect char $p > 0$

$V = \operatorname{Spec} K$ $K_0(V, \bar{\mathbb{Q}}_\ell)$ $\Vert$

abs. Galois. $K_0(G_K, \bar{\mathbb{Q}}_\ell) \xrightarrow{Sw} \mathbb{Z}$ Swan conductor

conductor formula.

$$Sw_K H^*_c(U_{\bar{K}}, \mathcal{F}) = \operatorname{rk} \mathcal{F} \cdot Sw_K H^*(U_{\bar{K}}, \mathbb{Q}_\ell) - \deg Sw \mathcal{F}$$

Swan class: Certain 0-cycle class of the reduction of a proper model of $U / \mathcal{O}_K$

~~Generalization~~

~~Analogue of a gen' of the GOS-formula~~

~~$\chi_c(U_{\bar F}, \mathcal{F}) = \mathrm{rk}\,\mathcal{F} \cdot \chi_c(U_{\bar F}, \mathbb{Q}_\ell) - \deg \mathrm{Sw}\,\mathcal{F}$~~

~~U/k smooth $\mathrm{ch}\,k = p > 0$ $\mathcal{F}$ perfect $\mathcal{F}/U$ smooth~~

Generalization of the conductor formula

- ~~constructible sheaf~~
- ~~relative version~~

$f : U \to V$.

$$\begin{array}{ccc} f_! : K_0(U, \bar{\mathbb{Q}}_\ell) & \longrightarrow & K_0(V, \bar{\mathbb{Q}}_\ell) \\ \downarrow \overline{\mathrm{Sw}}_U & & \downarrow \overline{\mathrm{Sw}}_V \\ F_0 G(\partial \bar U_F)_{\mathbb{Q}} & \longrightarrow & F_0 G(\bar V_F)_{\mathbb{Q}} \end{array}$$

modification of Swan class

K-theoretic version of the gp of 0-cycle cl. supported on the closed fiber of the compactification of U / O_K

Main ingredients

1. Definition of Swan class
   (a) smooth case K-th. loc. int. product
   (b) constructible case Excision
2. Pf of the formula
   Devissage : reduction to • induction formula (finite covering)
   • relative curve

   logarithmic Lefschetz trace formula
   open variety, σ ~~(wild~~ $\hat{P}_K$ G_{F_F} wild inertia

3) Def'n of the Swan class

Classical case = Swan conductor

$G = \mathrm{Gal}(L/K)$ ~~totally ramified~~ finite Galois ext

M rep of G

$$\mathrm{Sw}_K M = \frac{1}{|G|} \sum_{\substack{\sigma \neq 1 \\ \sigma \in G}} s_G(\sigma)\left(\mathrm{Tr}(\sigma : M) - \dim M\right)$$

$$s_G(\sigma) = -v_L\left(\frac{\sigma(\pi_L)}{\pi_L} - 1\right)$$

$f : V \to U$ finite étale G-torsor

of smooth sep'd sch of f.t./K. $\sigma \in G, \neq 1$

$$((\Gamma_\sigma, \Delta_V))$$

First approximation Geom. case /F

$V \hookrightarrow Y$ smooth cpt'n
$\downarrow$
U

$$(\bar{\Gamma}, \Delta_Y) \in CH_0(Y \setminus V)$$

Corrections $\Omega(\log)$

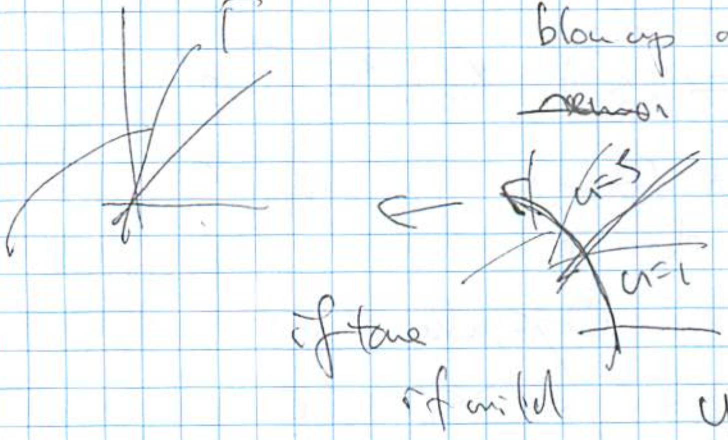
1. log version. - put log on the bdry. ~~[illegible]~~ focus on wild ramif'n. not Artin but Swan
2. Alteration -
3. limit of compactification
4. over $S = \mathrm{Spec}\, O_K$. replace Chow gp by K-theory

4

1. $V \subset Y \supset D = \bigcup D_i$ div. S.N.C.

replace $Y \times Y$ by log product

blow up at every $D_i \times D_i$



if tame

if wild

$u = \frac{1 \otimes t}{t \otimes 1}$

$\xi^{\,n} = 1$

$t \otimes 1 = \xi \otimes$ ... $V \hookrightarrow Y$, $U \hookrightarrow X$, $t^n = s$

$\frac{\quad}{1 \otimes t} = \xi$

remove prop. transform.

$$(\Gamma, \Delta_Y^{\log})_{(Y \times Y)^\sim}$$

2. alteration

$$\begin{array}{ccc} W & \hookrightarrow & Z \\ g \downarrow & & g \downarrow \\ V & \hookrightarrow & Y \end{array} \qquad \frac{1}{[Z:Y]} g_*((g \times g)^* \Gamma, \Delta_Z)_{(Z \times Z)^\sim}$$

3. $\varprojlim_Y CH_0(Y \setminus V)$ No previleged compactificat.

4. replace $CH_0(Y \setminus V)$ by

$$F_0 G(Y_F)$$

$$((\Gamma_f, Y))_{(Y \times_S Y)^\sim} = (-1)^d \left([\mathrm{Tor}_d^{\mathcal{O}_{(Y \times_S Y)^\sim}}(\mathcal{O}_\Gamma, \mathcal{O}_\Delta)] \right.$$

for $d >> 0$

$$\left. - [\mathrm{Tor}_{d+1}(\text{—})] \right)$$

is in —. indep of d, defined even for $\sigma = 1$

Swan class $\quad f: V \to U \quad \Gamma \subset G \leftrightarrow \mathcal{F}$

$$Sw_U \mathcal{F} = \frac{1}{|G|} \sum_{\substack{\sigma \in G \\ \neq 1}} f_! ((\Gamma_\sigma, \Delta_V)) (\mathrm{Tr}(\sigma : M) - \mathrm{rk}\, \mathcal{F})$$

$$\in F_0 G(\cancel{\ } \overline{U})_{\mathbb{Q}}$$

- total Swan class

$$\overline{Sw}_U \mathcal{F} = \frac{-1}{|G|} \sum_{\sigma \in G} f_! ((\Gamma_\sigma, \Delta_V)) \mathrm{Tr}(\sigma : M)$$

$$= \overline{Sw_U \mathcal{F}} \cancel{\ } \mathrm{rk}\, \mathcal{F} \cdot ((\Delta_U, \Delta_U))$$

- Constructible case

$U_1 \subset U \supset U_0 \qquad \mathcal{F}|_{\cancel{\ }}$ smooth on U

smooth closed subsch

Excision formula

$\Rightarrow \quad \underline{Sw}_U \mathcal{F} = \underline{Sw}_{U_0} \mathcal{F}|_{U_0} + \underline{Sw}_{U_1} \mathcal{F}|_{U_1}$

and

$\Rightarrow \quad Sw, \overline{Sw}$ def'd for constr.

R - R

$f: U \to V \quad$ ~~sep~~ morph of sep. sch of f.t $/k$

$\mathcal{F}$ const. sh. on $U \Rightarrow$

$$f_! \overline{Sw}_U \cancel{\ } \mathcal{F} = \overline{Sw}_V R f_! \mathcal{F}.$$

$V = \mathrm{Spec}\, k \Rightarrow$ conductor formula.

log Lefschetz trace formula

L/K finite ramified Galois ext

$\sigma \in P \subset G = G_{L/K}$

$D \subset X \supset U / \mathcal{O}_L$ $D' \subset X' \supset U'$ conj by σ

$\Gamma' \subset (X_L \times_L X'_L)'$ s.t

$\Gamma' \cap (D_L \times X'_L)' \subset \Gamma' \cap (X_L \times D'_L)$

$\Gamma = (U_L \times U'_L) \cap \Gamma'$

$\Rightarrow pr_2 : \Gamma \longrightarrow U_L$ proper

$\mathrm{Tr}(\Gamma_{\sigma}^{*} : H_c^*(U_{\bar{L}}, \mathbb{Q}_\ell))$

$= \deg(\Delta_{X_t}, \Gamma_t')$