教授 (Professor)

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A. 研究概要

剰余標数が $p > 0$ の完備離散付値体のガロワ群の上つき分岐群によるフィルトレーションを調べた．離散付値体の接空間を構成し，接空間の射が支配的となる拡大でフィルトレーションが保たれることを証明した．このことを使って，剰余体が完全体という古典的な場合に帰着させることにより，分岐群の次数商が $\mathbf{F}_p$ 線形空間であることを証明した．同じ方法により，ガロワ表現の導手が整数であるという Hasse–Arf の定理を剰余体が一般の場合に拡張した．分岐群の次数商が $\mathbf{F}_p$ 線形空間であることの，幾何的な方法による別の証明も与えた．この方法により，次数商の指標群から接空間の双対空間への単射を構成した．この構成により，整数環上の有限型の正則スキーム上の ℓ 進層に対し，その特性サイクルを各閉ファイバーの密開集合上で定義できる．以上の成果について，論文を準備中である．

I studied the filtration by upper ramification groups of Galois groups of a complete discrete valuation field of residue characteristic $p > 0$. I constructed the tangent space of discrete valuation field and proved that the filtration is preserved for extensions such that the morphism on the tangent spaces is dominant. Using this property, I proved that the graded quotients of ramification groups are $\mathbf{F}_p$-vector spaces by reduction to the classical case where the residue field is perfect. By the same method, the Hasse-Arf theorem asserting the integrality of the conductor of Galois representations is generalized to arbitrary residue field case. I gave another geometric proof of the theorem that the graded quotients of ramification groups are $\mathbf{F}_p$-vector spaces. This proof gives an injection from the character groups of graded quotients to twisted duals of the tangent space. This construction defines the characteristic cycle of an ℓ-adic sheaf on a regular scheme of finite type over the integer ring, on a dense open of each geometric closed fiber. I am preparing articles on the results.

B. 発表論文

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2. K. Kato, I. Leal, T. Saito “Refined Swan conductors mod p of one-dimensional Galois representations”, Nagoya Mathematical Journal 236 (2019), 134–182.
3. T. Saito “Ramification groups of coverings and valuations”, Tunisian Journal of Mathematics Vol. 1, No. 3, 373-426, 2019
4. T. Saito “On the proper push-forward of the characteristic cycle of a constructible sheaf”, Proceedings of Symposia in Pure Mathematics Volume: 97; 2018; Algebraic Geometry: Salt Lake City 2015, Part 2, 485-494
5. T. Saito, Y. Yatagawa “Wild ramification determines the characteristic cycle”, Annales Scientifiques de l’Ecole normale superieure, 50, fascicule 4 (2017), 1065-1079.
6. T. Saito “Characteristic cycle of the external product of constructible sheaves”, Manuscripta Mathematica, 154, Issue 1-2, 2017, pp 1-12.
7. T. Saito “Wild ramification and the cotangent bundle”, Journal of Algebraic Geometry, 26 (2017), 399-473.
8. T. Saito “The characteristic cycle and the singular support of a constructible sheaf”, Inventiones mathematicae, 207(2), (2017) 597-695,
9. T. Saito “Characteristic cycle and the Euler number of a constructible sheaf on a surface”, Kodaira Centennial issue of the Journal of Mathematical Sciences, the University of Tokyo, vol 22, (2015), 387-442.

C. 口頭発表

1. Graded Quotients of Ramification Groups of a Local Field with Imperfect Residue Field, January 7, 2020, International

conference on arithmetic geometry 2020, TIFR, Mumbai.（インド）mercredi 22 janv. 2020, IHES.（フランス）

2. Etale Cohomology and the Characteristic Cycle, September 6, 2019, BICMR, Pekin University.（中国）
3. Ramification groups of a local field (with Ahmed Abbes and Kazuya Kato), September 5, 2019 CAS Beijing.（中国）
4. CC, Wild Ramification and Irregular Singularities, Sep 25, 2019 at IMPAN in Warsaw, Poland.（ポーランド）
5. Characteristic cycle of a constructible sheaf, Arithmetic Geometry in Carthage, Summer School, Tunisian Academy Beit al-Hikma, Carthage, Tunisia Thursday, June 20-21, 2019.（チュニジア）
6. Characteristic cycle of constructible sheaves and restriction to curves. "Arithmétique et géométrie algébrique", une conférence en l'honneur d'Ofer Gabber, à l'occasion de son soixantième anniversaire, à l'IHÉS, Vendredi 15 juin, 2018.（フランス）Cohomology of algebraic varieties CIRM October 19th, 2018.（フランス）
7. Characteristic cycle of an étale sheaf and its functoriality, Purdue University, September 24-28, 2018.（アメリカ）
8. Characteristic cycles and the conductor of direct image, Interactions between Representation Theory and Algebraic Geometry, the University of Chicago, August 22, 2017（アメリカ）, p 進コホモロジーと数論幾何学， 東京電機大学 11 月 16 日, The Legacy of Carl Friedrich Gauss, Dec 18, 2017, TSIMF, Sanya,（中国）. Motives in Tokyo on the occation of Shuji Saito's 60th Birthday March 26, 2018, Univ. of Tokyo.
9. Characteristic cycle of an ℓ-adic sheaf, 数学会総合分科会, 特別講演，関西大学, 2016 年 9 月 17 日，東北大学代数セミナー 2017 年 1 月 26 日， 第 12 回 鹿児島 代数・解析・幾何学セミナー 2017 年 2 月 13 日，JAMI 2017 Local zeta functions and the arithmetic of moduli spaces: A conference in memory of Jun-Ichi Igusa March 22-26, 2017 Johns Hopkins University（アメリカ）, Fukuoka International Conference on Arithmetic Geometry in 2017 April 20, (日本). Workshop on arithmetic geometry at Tambara 2017 May 22,（日本）. Algebraic Analysis in honor of Masaki Kashiwara's 70th birthday IHES, June 9 2017（フランス）. Algebraic Analysis and Representation Theory – In honor of Professor Masaki Kashiwara's 70th Birthday – RIMS June 28（日本）. Regulators in Niseko 2017, 2017 年 9 月 4 日. Tokyo-Lyon Satellite Conference in Number Theory, Univ. of Tokyo, February 21 (Wed), 2018. 第 34 回京都賞記念ワークショップ「基礎科学部門」2018 年 11 月 12 日（月）京大数理研 Arithmetic and Algebraic Geometry 2019 - in honour of Professor Tomohide Terasoma's 60th birthday - January 25 (Fri), 2019 東大数理，CAS Beijing, September 4, 2019（中国）
10. On the characteristic cycle of a constructible sheaf, SAGA Orsay, 22/03/16,（フランス）. Hakodate workshop on arithmetic geometry 2016. Hakodate arena.（日本） 6 月 1 日. Tokyo-Seoul at KIAS, June 16-17, 2016（韓国）. PANT July 11-15 2016（台湾）.

D. 講義

1. 数理科学基礎＋微分積分学（通年）：微積分 (教養学部前期課程講義)
2. 全学自由ゼミナール：圏と層. (教養学部前期課程講義)

E. 修士・博士論文

1. (博士) 加藤 大輝 (KATO Hiroki): Wild ramication, the nearby cycle complexes, and the characteristic cycles of ℓ-adic sheaves.

F. 対外研究サービス

1. 第 23, 24 回高木レクチャー　6 月 8 日，12 月 8 日オーガナイザー
2. Arithmetic Geometry in Carthage June 17-28, 2019 Tunisian Academy Beit al-Hikma, Carthage, Tunisia オーガナイザー
3. Journal of Algebraic Geometry, エディター
4. Documenta Mathematica, エディター
5. Japanese Journal of Mathematics, エディター

G. 受賞

日本数学会出版賞：『数学の現在』共編．2019 年 3 月 18 日

H. 海外からのビジター

A. Abbes (IHES, CNRS 主任研究員)
大学院講義　"Flattening theorem following Raynaud-Gruson and Guignard"を行った．
講義概要　The goal of this cours is to present two proofs of the following theorem of Raynaud-Gruson, the original one and a new one by Quentin Guignard:

Theorem 1. *Let X be a coherent scheme (i.e., quasi-compact and quasi-separated), $f\colon Y \to X$ a morphism of finite presentation, U a quasi-compact open subset of X and $\mathcal{F}$ a quasi-coherent $\mathcal{O}_Y$-module of finite type. Assume that the restriction of $\mathcal{F}$ to $Y \times_X U$ is a finitely presented $\mathcal{O}_{Y\times_X U}$-module which is flat over U. Then, there exists a blow-up $\phi\colon X' \to X$ such that:*
(i) *The center of the blow-up ϕ is a finitely presented closed subscheme of X, disjoint from U.*
(ii) *If Y' is the strict transform of the X-scheme Y along ϕ, then the strict transform $\mathcal{F}'$ of $\mathcal{F}$ along ϕ is finitely presented over $\mathcal{O}_{Y'}$ and flat over X'.*

This theorem is intimately related to rigid geometry. A variant for formal schemes plays a crucial role for the study of flatness in rigid geometry, which was Raynaud's main motivation. Moreover, Guignard's new proof unravels important features of rigid varieties, particularly their local structure, shedding new lights on the theory of adic spaces. This course is for students who have a basic knowledge in algebraic geometry. No knowledge of rigid geometry is required as I will not discuss the formal/rigid variant of the theorem.
滞在中は，p 進 Hodge 理論における相対 Hodge-Tate スペクトル系列に関する研究を行った．