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A. 研究概要

剰余体が完全とは限らない完備離散付値体のガロワ群の分岐群によるフィルトレーションを，剰余体が完全な場合に帰着できることが昨年度分かったので，まずその論文を完成し投稿した．この結果を使うと，分岐群の次数商を剰余体の微分形式と結びつけることができる．また，剰余体が完全な場合に帰着させることによる，分岐群によるフィルトレーションの特徴づけも得られるので，その論文も完成し投稿した．

混標数のスキーム上のエタール層の特異台の定義のための最初の障害はこのようなスキーム上の余接束が定義されていないことである．微分の加法性，Leibniz 則をそれぞれ Witt ベクトルの加法を定義する多項式と，Frobenius 作用素を使って修正することにより，微分形式の加群の定義を修正して，Frobenius-Witt 微分形式の加群を定義した．さらに正則スキーム上では，この加群が標数 p のファイバー上の自由加群であることを証明した．このことを使って余接束の修正版を定義した．さらに層についてのある強い仮定のもとで，特異台が存在することを証明した．この 2 つの結果についてそれぞれ論文を完成し投稿した．

I found last year that the definition of filtration by ramification groups of Galois groups of a complete discrete valuation field with not necessarily perfect residue field is reduced to those with perfect residue field. As an application, the graded quotients are related to differential forms of the residue fields. I completed an article on this result and submitted to a journal. I also obtained a characterization of the filtration by the reduction to the perfect residue field case. This result is also written up and is submitted.

A first obstruction in the definition of the singular support of an étale sheaf on a scheme of mixed characteristic is the absense of the cotangent bundle. By modifying respectively the additivity by the polynomial defining the addition of Witt vectors and the Leibniz rule using the Frobenius, we define the module of Frobenius-Witt differentials as a modification of that of usual differentials. I proved that the module of Frobenius-Witt differentials on a regular scheme is locally free on the fiber of characteristic p. Using this result, I define a modification of the cotangent bundle on the fiber of characteristic p. Further, under a certain strong assumption on the sheaf, I proved the existence of the singular support. I completed two articles on these results and submitted to a journal.

B. 発表論文

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5. T. Saito "On the proper push-forward of the characteristic cycle of a constructible sheaf", Proceedings of Symposia in Pure Mathematics Volume: 97; 2018; Algebraic Geometry: Salt Lake City 2015, Part 2, 485-494

6. T. Saito, Y. Yatagawa "Wild ramification determines the characteristic cycle", Annales Scientifiques de l'Ecole normale superieure, 50, fascicule 4 (2017), 1065-1079.

7. T. Saito "Characteristic cycle of the external product of constructible sheaves", Manuscripta Mathematica, 154, Issue 1-2, 2017, pp 1-12.

8. T. Saito "Wild ramification and the cotangent bundle", Journal of Algebraic Geometry, 26 (2017), 399-473.

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C. 口頭発表

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2. Graded Quotients of Ramification Groups of a Local Field with Imperfect Residue Field, January 7, 2020, International conference on arithmetic geometry 2020, TIFR, Mumbai.（インド）mercredi 22 janv. 2020, IHES.（フランス）

3. Etale Cohomology and the Characteristic Cycle, September 6, 2019, BICMR, Pekin University. （中国）

4. Ramification groups of a local field (with Ahmed Abbes and Kazuya Kato), September 5, 2019 CAS Beijing.（中国）

5. CC, Wild Ramification and Irregular Singularities, Sep 25, 2019 at IMPAN in Warsaw, Poland.（ポーランド）

6. Characteristic cycle of a constructible sheaf, Arithmetic Geometry in Carthage, Summer School, Tunisian Academy Beit al-Hikma, Carthage, Tunisia Thursday, June 20-21, 2019.（チュニジア）

7. Characteristic cycle of constructible sheaves and restriction to curves. "Arithmétique et géométrie algébrique", une conférence en l'honneur d'Ofer Gabber, à l'occasion de son soixantième anniversaire, à l'IHÉS, Vendredi 15 juin, 2018.（フランス）Cohomology of algebraic varieties CIRM October 19th, 2018. （フランス）

8. Characteristic cycle of an étale sheaf and its functoriality, Purdue University, September 24-28, 2018. （アメリカ）

9. Characteristic cycles and the conductor of direct image, Interactions between Representation Theory and Algebraic Geometry, the University of Chicago, August 22, 2017 （アメリカ）, p進コホモロジーと数論幾何学， 東京電機大学 11 月 16 日, The Legacy of Carl Friedrich Gauss, Dec 18, 2017, TSIMF, Sanya, （中国）. Motives in Tokyo on the occation of Shuji Saito's 60th Birthday March 26, 2018, Univ. of Tokyo.

10. Characteristic cycle of an ℓ-adic sheaf, 数学会総合分科会，特別講演，関西大学，2016 年 9 月 17 日，東北大学代数セミナー 2017 年 1 月 26 日， 第 12 回 鹿児島 代数・解析・幾何学セミナー 2017 年 2 月 13 日, JAMI 2017 Local zeta functions and the arithmetic of moduli spaces: A conference in memory of Jun-Ichi Igusa March 22-26, 2017 Johns Hopkins University（アメリカ）, Fukuoka International Conference on Arithmetic Geometry in 2017 April 20, (日本). Workshop on arithmetic geometry at Tambara 2017 May 22, （日本）. Algebraic Analysis in honor of Masaki Kashiwara's 70th birthday IHES, June 9 2017（フランス）. Algebraic Analysis and Representation Theory – In honor of Professor Masaki Kashiwara's 70th Birthday – RIMS June 28 （日本）. Regulators in Niseko 2017, 2017 年 9 月 4 日. Tokyo-Lyon Satellite Conference in Number Theory, Univ. of Tokyo, February 21 (Wed), 2018. 第 34 回 京都賞記念ワークショップ「基礎科学部門」2018 年 11 月 12 日（月）京大数理研 Arithmetic and Algebraic Geometry 2019 - in honour of Professor Tomohide Terasoma's 60th birthday - January 25 (Fri), 2019 東大数理，CAS Beijing, September 4, 2019 (中国)

D. 講義

1. 数理科学基礎＋微分積分学（通年）：微積分（教養学部前期課程講義）

E. 修士・博士論文

1. (博士) 竹内 大智 (TAKEUCHI Daichi): On

the epsilon factors of ℓ-adic sheaves on varieties.

2. (修士) 今井 湖都 (IMAI Koto): Ramification groups of some finite Galois extensions of maximal nilpotency class over local fields of positive characteristic.

3. (修士) 吉田 匠 (YOSHIDA Takumi) On the 2-part of Birch-Swinnerton-Dyer conjecture for elliptic curves with complex multiplication by the ring of integers of $\mathbf{Q}(\sqrt{-7})$.

F. 対外研究サービス

1. Journal of Algebraic Geometry, エディター

2. Documenta Mathematica, エディター

3. Japanese Journal of Mathematics, エディター

G. 受賞
日本数学会出版賞：『数学の現在』共編. 2019 年 3 月 18 日