Logarithmic transformations of rigid analytic elliptic surfaces

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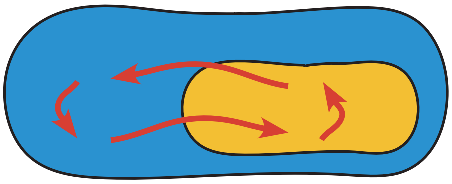
§1. Analytification, Algebraization, and GAGA.

k : algebraically closed non-Archimedean complete valuation field

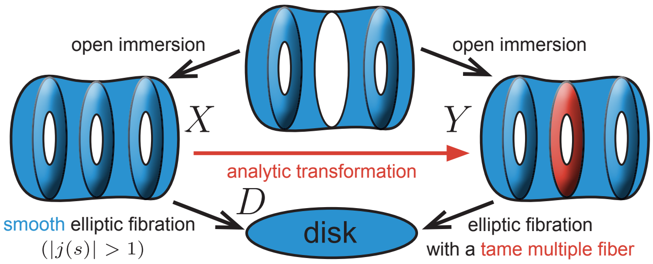


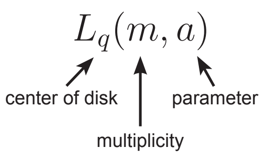
category of proper rigid analytic spaces

category of proper schemes

logarithmic transformations

§2. Logarithmic transformations.





$L_q(m,a)$ does not change the fibration outside the central fiber.

The complex analytic case was given by Kodaira.

§3. Examples given by logarithmic transformations.

Algebraic surfaces in any characteristic:

Hyperelliptic surfaces,

Quasi-hyperelliptic surfaces, etc.

Non-algebraic surfaces in any characteristic:

Analytic surfaces with pathological invariants in positive characteristics.

§4. Invariants of resulting surfaces.

E : Tate's elliptic curve C: proper smooth curve

$$(Y\to C):=(L_q(m_q,a_q))_q(E\times C\to C)$$

$$u:=\sharp\{q\mid p \text{ divides } m_q\}\qquad \eta:=\sum_q \frac{a_q}{m_q}\in k$$

Theorem. $\chi(\mathcal{O}_Y)=0,$

$$h^0(\Omega^1_Y)=\begin{cases} g(C)+u-1, & u\neq 0,\\ g(C), & u=0 \text{ and } \eta\neq 0,\\ g(C)+1, & u=0 \text{ and } \eta=0.\end{cases}$$

§5. Construction by two methods.

Analytic method: Rigid analytic quotient of actions on relative Tate's uniformizations.

This method gives the diagram in §2.

Algebraic method: Quotient of finite actions on elliptic fibrations.

This method gives the same fibration as in the above method.

Relative Tate's uniformization:

$$X\cong \mathbb{G}_{m,D}/\langle q\rangle\ (q\in\mathcal{O}_D(D),\ 0<|q(s)|<1)$$

Finite coverings and Equivariant actions:

$$D'/D,\ \mu_m\times D'\to D',\ (\zeta,s)\mapsto \zeta s$$

$$X':=X\times_D D',\ X'[m]\cong \mathbb{Z}/m\mathbb{Z}\times\mu_m$$

$$X'/Y,\ \mu_m\times X'\to X',\ (\zeta,x,s)\mapsto(\zeta^a x,\zeta s)$$

Remarks: In the complex analytic case, we can use actions of $X'[m]$. In the rigid analytic case, we use only actions of $\mu_m\subset X'[m]$. The other actions do not give the diagram in §2.

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