

**Berkeley–Tokyo workshop  
on Number theory and Arithmetic geometry**

March 15–17 Tokyo and March 14–16 Berkeley, online (registration required)

**Program**

March 15 Tuesday Tokyo/14 Monday Berkeley

9:15–10:15 Tokyo /17:15–18:15 Berkeley

Dong Gyu Lim (UC Berkeley)

Nonemptiness of (single) affine Deligne-Lusztig varieties

10:30–11:30 Tokyo /18:30–19:30 Berkeley

Yasuhiro Oki (U Tokyo)

On the connected components of Shimura varieties for CM unitary groups in odd variables

March 16 Wednesday Tokyo/15 Tuesday Berkeley

9:15–10:15 Tokyo /17:15–18:15 Berkeley

Yanshuai Qin (UC Berkeley)

On the Brauer groups of arithmetic schemes

10:30–11:30 Tokyo /18:30–19:30 Berkeley

Yuki Yamamoto (U Tokyo)

Comparing Bushnell–Kutzko and Sécherre’s constructions of types for  $GL_N$  and its inner forms with Yu’s construction

March 17 Thursday Tokyo/16 Wednesday Berkeley

9:15–10:15 Tokyo /17:15–18:15 Berkeley

Owen Barrett (UC Berkeley)

The derived category of the abelian category of constructible sheaves

10:30–11:30 Tokyo /18:30–19:30 Berkeley

Tepei Takamatsu (U Tokyo)

On the Shafarevich conjecture for irreducible symplectic varieties

This is a workshop in the framework of the Strategic Partnerships Project of the University of Tokyo.

Organizers:

Yoichi Mieda, Martin Olsson, Takeshi Saito, Sug Woo Shin.

## Titles and Abstracts

### Owen Barrett

Title: The derived category of the abelian category of constructible sheaves

Abstract: Nori proved in 2002 that given a complex algebraic variety  $X$ , the bounded derived category of the abelian category of constructible sheaves on  $X$  is equivalent to the usual triangulated category  $D_c^b(X)$  of bounded constructible complexes on  $X$ . He moreover showed that given any constructible sheaf  $F$  on  $\mathbf{A}^n$ , there is an injection  $F \hookrightarrow G$  with  $G$  constructible and  $H^i(\mathbf{A}^n, G) = 0$  for  $i > 0$ .

In this talk, I'll discuss how to extend Nori's theorem to the case of a variety over an algebraically closed field of positive characteristic, with Betti constructible sheaves replaced by  $\ell$ -adic sheaves. This is the case  $p = 0$  of the general problem which asks whether the bounded derived category of  $p$ -perverse sheaves is equivalent to  $D_c^b(X)$ , resolved affirmatively for the middle perversity by Beilinson.

### Dong Gyu Lim

Title: Nonemptiness of (single) affine Deligne-Lusztig varieties

Abstract: In the study of Shimura varieties, it is an important problem to count the points reduction modulo  $p$  (Langlands-Rapoport conjecture) as it provides a way to compute the Hasse-Weil zeta function. The most interesting piece showing up in the point counting is affine Deligne-Lusztig variety (ADLV) and it has been studied in various level structures including the hyperspecial level and the Iwahori level. In the talks, we will see explicit examples of ADLV described as a set of certain lattices and flags in the modular curve case. Then we will discuss the nonemptiness criterion for a single ADLV along with the results already known and newly discovered. If time permits, the dimension formula will be discussed shortly.

### Yasuhiro Oki

Title: On the connected components of Shimura varieties for CM unitary groups in odd variables

Abstract : Let  $(G, X)$  be a Shimura datum. Take a prime number  $p$  and a Bruhat-Tits subgroup  $K_p$  of  $G(\mathbb{Q}_p)$ . Consider the projective limit of the sets of connected components of Shimura varieties for  $(G, X)$  whose level at  $p$  are given by  $K_p$ . It is equipped with the prime-to- $p$  Hecke action. Then we discuss the question whether the above action is transitive, which is motivated by the theory of mod  $p$  reductions of Shimura varieties. In this talk, we give infinitely many projective systems of the Shimura varieties for CM unitary groups in odd variables

for which the considered question is negative. To achieve this goal, we study a question related to the weak approximation on certain tori over  $\mathbb{Q}$ .

### **Yanshuai Qin**

Title: On the Brauer groups of arithmetic schemes

Abstract: Let  $X$  be a regular scheme projective flat over the ring of integers in a number field  $K$ , we prove a relation between the Brauer group of  $X$ , the geometric Brauer group of  $X_K$  and the Tate-Shafarevich group of  $\text{Pic}_{X/K}^0$ , generalizing a theorem of Artin and Grothendieck for arithmetic surfaces to arbitrary dimensions. As a result, we reduce Artin's question about the finiteness of  $\text{Br}(X)$  for arithmetic schemes to arithmetic 3-folds.

### **Teppei Takamatsu**

Title: On the Shafarevich conjecture for irreducible symplectic varieties

Abstract: The Shafarevich conjecture, which was proved by Faltings and Zarhin, states the finiteness of isomorphism classes of abelian varieties of a fixed dimension over a fixed number field admitting good reduction away from a fixed finite set of finite places. In this talk, we prove the Shafarevich conjecture for irreducible symplectic varieties (= hyper-Kähler varieties) whose second Betti numbers are greater than 4. This talk is based on joint work with Lie Fu, Zhiyuan Li, and Haitao Zou.

### **Yuki Yamamoto**

Title: Comparing Bushnell–Kutzko and Sécherre's constructions of types for  $\text{GL}_N$  and its inner forms with Yu's construction

Abstract: Let  $A$  be a finite-dimensional central simple algebra over a non-archimedean local field, and let  $G$  be the multiplicative group of  $A$ . When we consider smooth complex representations of  $G$ , there exist several constructions of types for supercuspidal representations of  $G$ . In this talk, we compare two constructions of types, maximal simple types by Sécherre and Yu's construction of types for  $G$ . In particular, we show that tame supercuspidal representations for  $G$  defined by Yu are essentially tame in the sense of Bushnell–Henniart. This talk is based on joint work with Arnand Maxeux.