

Third International Research Institute of Mathematical Society of Japan

GEOMETRIC COMPLEX ANALYSIS



editors

J. Noguchi, H. Fujimoto, J. Kajiwara & T. Ohsawa

Third International Research Institute of Mathematical Society of Japan

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Hayama, Japan 19–29 March 1995

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PREFACE

This proceedings volume consists of papers by participants of the conference

Geometric Complex Analysis

which was held at Hayama, March 20–29, 1995. This was the third in the series of International Research Institute of the Mathematical Society of Japan. We asked all the participants who gave talks at the meeting to send abstracts and to submit papers. All the manuscripts were refereed and not all were accepted.

For financial support, our thanks are due to the Mathematical Society of Japan, Tokyo Institute of Technology, the Commemorative Association for the Japan World Exposition (1970), Village Shonan Inc., and the Mitsubishi Trust Yamamuro Memorial Scholarship Foundation. Thanks are also due to Shonan Village Center for its hospitality, and World Scientific Publishing Company for providing the publication, and most of all, to the participants in the conference who gave invited lectures and seminar talks. We are glad that so many branches of mathematics are in close relationship with complex analysis.

As mathematicians studying complex analysis in several variables, we are particularly glad to know that the spirit of magnificent works of Kiyoshi Oka is still vivid in this field. On this opportunity, we would like to evoke one of his significant remarks on his own work.

Le champs de fonctions analytiques de variables quelconques s'étend aux champs de arithmétique, algèbre, analyse, géométrie, et sciences exactes. C'est un fait, très simple mais tout fondamental. On rêvera aux nouveaux problèmes qui y s'attachent. C'est une des raisons que nous avons commencé à étudier la théorie des fonctions analytiques de plusieurs variables.

— quoted from “Note sur les fonctions analytiques de plusieurs variables” that appeared in *Kōdai Math. Sem. Rep. Nos. 5–6, Dec., 1949.*

December 1995

J. Noguchi
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Atsuji, A.

The second main theorem in Nevanlinna theory.

Bedford, E.

Dynamics of polynomial diffeomorphism of \mathbf{C}^2 .

Bland, J.

A variational problem for Hermitian Finsler metrics.

Frankel, S.

Uniformizing representations of surface groups into $\text{Diff}(S^1)$.

Hwang, A.

Extremal Kähler metrics minimize the Calabi energy (in their Kähler class).

Ivashkovich, S.

Extension properties of meromorphic mappings.

Kohn, J.

Global regularity.

Koo, H.

Carlson measure characterizations of BMOA on pseudoconvex domains.

Lempert, L.

The Virasoro group and complex analysis on infinite dimensional manifolds.

Nishimura, Y.

Iteration of some birational polynomial quadratic maps of \mathbf{P}^2 .

Schumacher, G.

Properties of Petersson-Weil metric.

Sibony, N.

Rational convexity and extension theorem of Ohsawa-Takegoshi currents.

Siu, Y.-T.

Hyperbolicity of complements of plane curves.

Tsuji, H.

On the global generation of adjoint bundles.

Wong, P.-M.

The second main theorem in Nevanlinna theory.

Wong, P.-M.

On hyperbolic manifolds.

Xing, Y.

Continuity of the complex Monge-Ampère operator.

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The purpose of this article is to present recent results of the author, which are extensions of the theorem of Lefschetz to the noncompact case. In this article we shall state the results, new ideas and sketches of proof. For more details, we refer the reader to the original paper[4].

The theorem of Lefschetz is as follows:

Let A be an abelian variety and let L be a positive line bundle over A . Then $H^0(A, \mathcal{O}(L^{\otimes i}))$ gives an embedding of A into a complex projective space \mathbb{P}^N if $i \geq 3$.

We extend this theorem to quasi-abelian varieties. A quasi-abelian variety is a toroidal group satisfying the generalized Riemann conditions. First we give the definition of a toroidal group.

Definition 1. A connected complex Lie group G without nonconstant holomorphic functions is called a toroidal group.

By a result of Morimoto, a toroidal group G is commutative. Then G is the quotient \mathbb{C}^n/Γ of \mathbb{C}^n by a discrete subgroup Γ . Furthermore, $\text{rank } \Gamma = n + m$, $1 \leq m \leq n$, and Γ has n elements which are linearly independent over \mathbb{Q} . Of course, it is a complex torus if $m = n$. Let \mathbb{R}_Γ^{n+m} be the real linear subspace of \mathbb{C}^n spanned by Γ . We denote by $\mathbb{C}_\Gamma^n = \mathbb{R}_\Gamma^{n+m} \cap \sqrt{-1}\mathbb{R}_\Gamma^{n+m}$ the maximal complex linear subspace contained in \mathbb{R}_Γ^{n+m} .

Definition 2. A toroidal group $G = \mathbb{C}^n/\Gamma$ is said to be a quasi-abelian variety if there exists a hermitian form \mathcal{H} on \mathbb{C}^n such that

- \mathcal{H} is positive definite on \mathbb{C}_Γ^n ,
- the imaginary part $\mathcal{A} := \text{Im } \mathcal{H}$ is integral valued on $\Gamma \times \Gamma$.