Title: *Computation of higher FR-torsion using the Framing Principle*

May 17 (Fri.) 15:00 - 16:00, 16:20 - 17:20

Suppose that we have a smooth bundle $E \to B$ with fiber M_t over $t \in B$ and a fiberwise generalized Morse function (GMF) $f_t : M_t \to \mathbb{R}$. Then by the usual Morse theory one obtains a family of based free chain complexes C_t. If $\pi_1 B$ acts trivially on $H_*(M)$ then we get an associated higher FR-torsion invariants $\tau_{2k}(f) \in H^{4k}(B;\mathbb{R})$ according to the theory we developed. The *Framing Principle* gives a formula for how this invariant depends on the choice of f_t. The expression for the cohomology class which is independent of the choice of f looks like this:

$$\tau_{2k}(E) = \tau_{2k}(f) + \sum_{i \geq 0} (-1)^{i+k} \zeta(2k+1) p_* ch_{4k}(\gamma_f^i).$$

I.e., it the correction term is an alternating sum of values of the Riemann zeta function times the push-down of the chern character in degree $4k$ of certain vector bundles γ_f^i along the Morse points of f of index i.

In this lecture I will show how this formula can be used to compute the invariant $\tau_{2k}(E)$ in many cases. For example, if the fibers are even dimensional then $\tau_{2k}(-f) = -\tau_{2k}(f)$ so $\tau_{2k}(E)$ is the average of the correction terms for f and $-f$. There are other cases, such as *Hatcher's example*, where $\tau_{2k}(f) = 0$ so the higher FR-torsion is equal to the correction term.

Another example is the following formula for the higher FR-torsion of the canonical surface bundle over the classifying space of the Torelli group T_g^s.

$$\tau_{2k}(T_g^s) = (-1)^k \zeta(2k+1) \frac{\kappa_{2k}}{2(2k)!}$$

where κ_{2k} is the degree $4k$ Miller-Morita-Mumford class.

In order to extend this to the κ_{odd} cases (in degree $4k+2$) and to find the lowest dimensional example of the phenomenon, I invented the *complex torsion invariants* $\tau_k^{\mathbb{C}}(E, m)$ for complex manifold bundles which satisfy a *Complex Framing Principle* and obtained the formula

$$\tau_k^{\mathbb{C}}(M_g^s, m) = -m^k \Re \left(\frac{1}{i^k} \mathcal{L}_{k+1}(\zeta_m) \right) \frac{\kappa_k}{k!}$$

where $\zeta_m = e^{2\pi i/m}$ and $\mathcal{L}_{k+1}$ is the classical polylogarithm.

This final formula implies that there is a *picture* for the Steinberg group of the cyclotomic number field $\mathbb{Q}(\zeta)$ dual to the first Miller-Morita-Mumford class κ_1. This brings us back to the beginning of the first lecture.