Title: “Review of Waldhausen K-Theory and Applications”
May 15 (Wed.) 15:00 - 16:00, 16:20 - 17:20
abstract

Waldhausen's $S_\bullet$-*construction* is something like a group completion for *Waldhausen categories* which are categories with certain types of admissible monomorphisms and weak equivalences. Two well-known examples are the category of finite sets and monomorphism and the category of finitely generated (f.g.) projective R-modules and splittable monomorphisms. Waldhausen's construction applied to these two categories gives the spaces $\Omega^\infty S^\infty(S^0)$ (or $\Omega^\infty S^\infty(X_+)$ if we take finite sets over a space X, i.e., with mappings to X) and the algebraic K-theory space $K_0R \times BGL(R)^+$. Weak equivalences are isomorphisms in both examples.

A Morse function f on a compact manifold M gives objects of both kinds. The critical set of f is a finite set over M (and thus over $B\pi_1 M$) and the associated cellular chain complex is a f.g. free (and thus projective) $\mathbb{Z}[\pi_1 M]$-complex. We combine these to form one Waldhausen category of based free *acyclic* chain complexes over a ring R with a specified group of units G. The main theorem (of John Klein and myself) is that the Waldhausen K-theory of this category is the homotopy fiber of the mapping

$$\Omega^\infty S^\infty(BG_+) \to \mathbb{Z} \times BGL(R)^+.$$

In this lecture I will go over the basic definitions and theorems (but not the proofs) of Waldhausen K-theory and show how it applies to Morse theory (as outlined above). One of the key points is to obtain an acyclic chain complex as required by the theory. I will also review the *framed function theorem* (from Monday's lecture) which allows us to construct a canonical fiberwise generalized Morse functions for any smooth fibration $M \to E \to B$. One application is the association of K-theory classes to the Torelli group.

Reference:

Waldhausen, Friedhelm, *Algebraic K-theory of spaces*, Algebraic and geometric topology (New Brunswick, N.J.,1983), Springer, Berlin, 1985, 318-419.